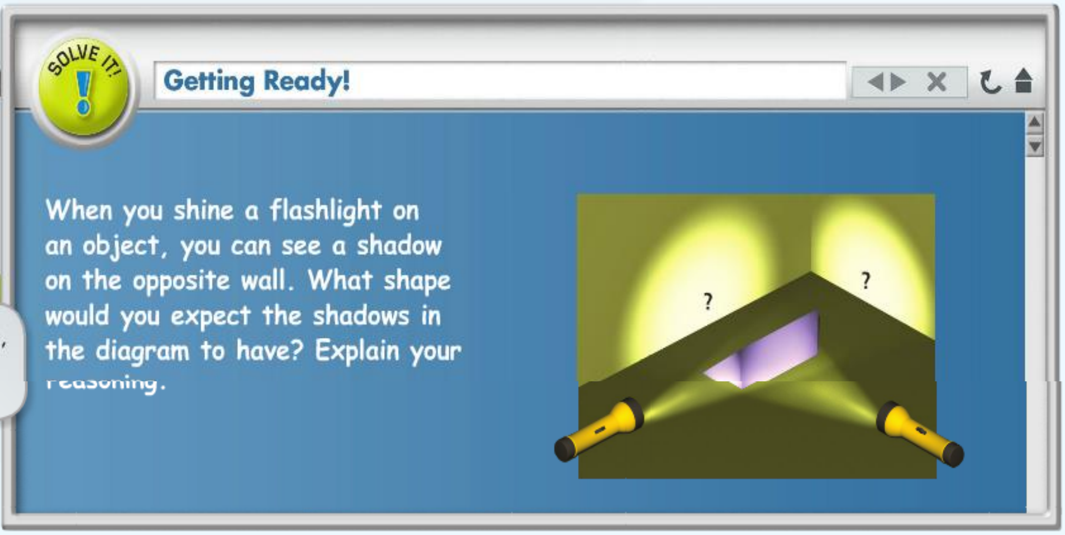
1-1 Nets and Drawings for Visualizing Geometry

Objective To make nets and drawings of three-dimensional figures

The object is 3-D, but its shadows are flat.



When you shine a flashlight on an object, you can see a shadow on the opposite wall. What shape would you expect the shadows in the diagram to have? Explain your reasoning.

Lesson Vocabulary
- net
- isometric drawing
- orthographic drawing

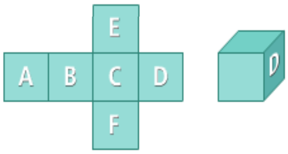
In the Solve It, you had to "see" the projection of one side of an object onto a flat surface. Visualizing figures is a key skill that you will develop in geometry.

Essential Understanding You can represent a three-dimensional object with a two-dimensional figure using special drawing techniques.

A **net** is a two-dimensional diagram that you can fold to form a three-dimensional figure. A net shows all of the surfaces of a figure in one view.

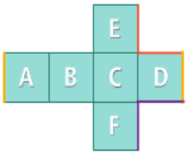
Problem 1 Identifying a Solid From a Net

The net at the right folds into the cube shown beside it. Which letters will be on the top and front of the cube?

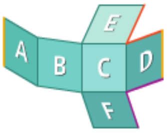


Think

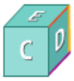
How can you see the 3-D figure? Visualize folding the net at the seams so that the edges join together. Track the letter positions by seeing one surface move in relation to another.



A, C, E, and F all share an edge with D when you fold the net, but only two of those sides are visible in the cube shown.



A wraps around and joins with D to become the back of the cube. B becomes the left side. F folds back to become the bottom.



E folds down to become the top of the cube. C becomes the front.

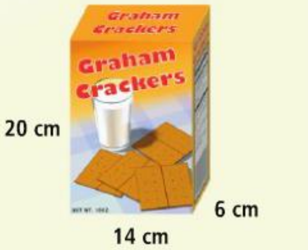
Got It? **1.** The net in Problem 1 folds into the cube shown at the right. Which letters will be on the top and right side of the cube?

Packaging designers use nets to design boxes and other containers like the box in Problem 2.

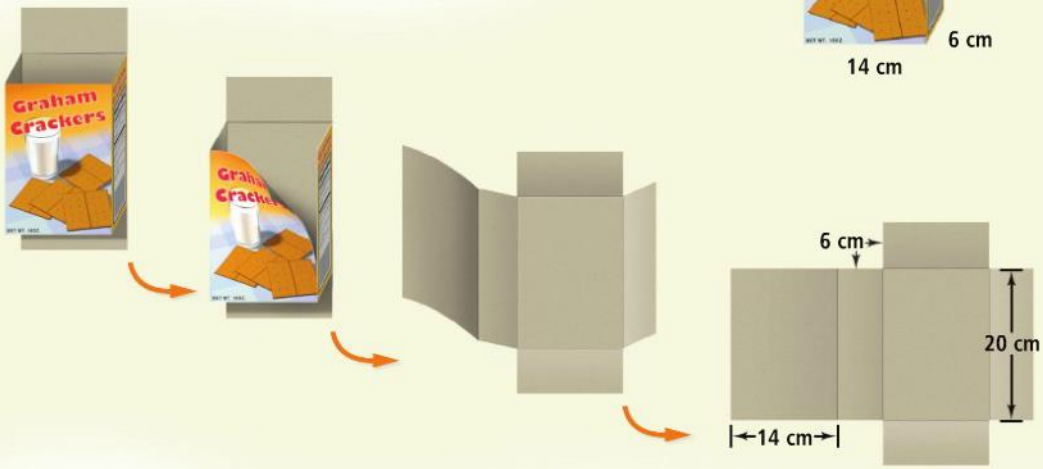
Think

How can you see the net?
Visualize opening the top and bottom flaps of the box. Separate one of the side seams. Then unfold and flatten the box completely.

Problem 2 Drawing a Net From a Solid

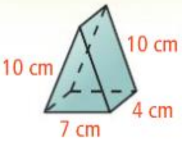


Package Design **What is a net for the graham cracker box to the right? Label the net with its dimensions.**



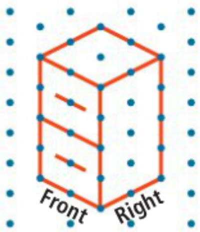
Got It? **2. a.** What is a net for the figure at the right? Label the net with its dimensions.

b. **Reasoning** Is there another possible net for the figure in part (a)? If so, draw it.



An **isometric drawing** shows a corner view of a three-dimensional figure. It allows you to see the top, front, and side of the figure. You can draw an isometric drawing on isometric dot paper. The simple drawing of a file cabinet at the right is an isometric drawing.

A net shows a three-dimensional figure as a folded-out flat surface. An isometric drawing shows a three-dimensional figure using slanted lines to represent depth.

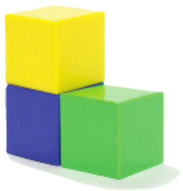


Plan

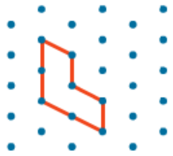
Is there more than one way to make an isometric drawing? Yes. You can start with any edge of the structure. Use that edge as a reference to draw the other edges.

Problem 3 Isometric Drawing

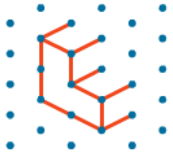
What is an isometric drawing of the cube structure at the right?



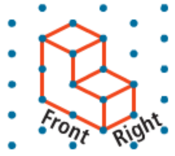
Step 1
Draw the front edges.



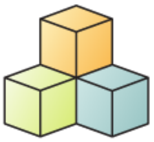
Step 2
Draw the right edges.



Step 3
Draw the back edges.



Got It? **3.** What is an isometric drawing of this cube structure?



An **orthographic drawing** is another way to represent a three-dimensional figure. An orthographic drawing shows three separate views: a top view, a front view, and a right-side view.

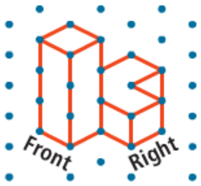
Although an orthographic drawing may take more time to analyze, it provides unique information about the shape of a structure.

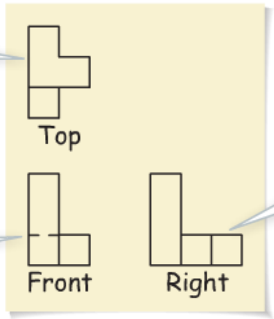
Plan

How can you determine the three views? Rotate the structure in your head so that you can "see" each of the three sides straight on.

Problem 4 Orthographic Drawing

What is the orthographic drawing for the isometric drawing at the right?



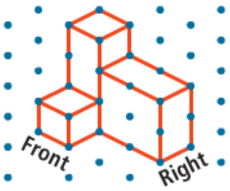


Solid lines show visible edges.

Dashed lines show hidden edges.

An isometric drawing shows the same three views.

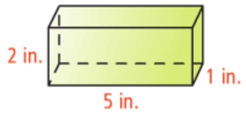
Got It? **4.** What is the orthographic drawing for this isometric drawing?



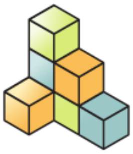
Lesson Check

Do you know HOW?

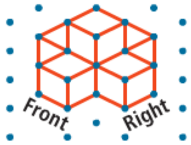
1. What is a net for the figure below? Label the net with its dimensions.



2. What is an isometric drawing of the cube structure?



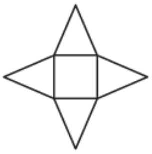
3. What is the orthographic drawing of the isometric drawing at the right? Assume there are no hidden cubes.



Do you UNDERSTAND?

4. **Vocabulary** Tell whether each drawing is *isometric*, *orthographic*, a *net*, or *none*.

a.



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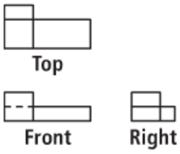
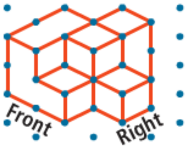
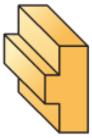


c.



d.



5. **Compare and Contrast** What are the differences and similarities between an isometric drawing and an orthographic drawing? Explain.

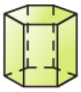
Practice and Problem-Solving Exercises

A Practice

Match each three-dimensional figure with its net.

See Problem 1.

6.



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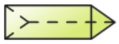

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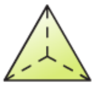
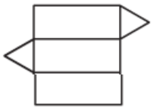


A.



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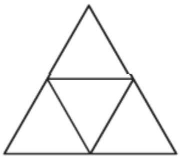

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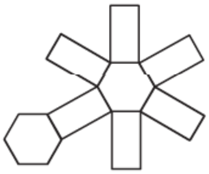
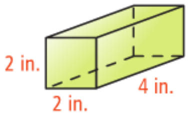


Draw a net for each figure. Label the net with its dimensions.

See Problem 2.

9.



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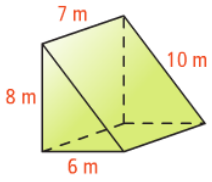

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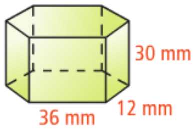
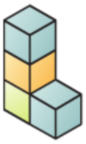


Make an isometric drawing of each cube structure on isometric dot paper.

See Problem 3.

12.



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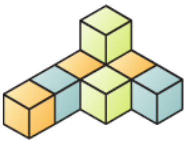

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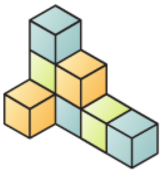

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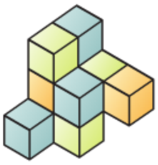
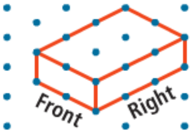


For each isometric drawing, make an orthographic drawing. Assume there are no hidden cubes.

See Problem 4.

16.



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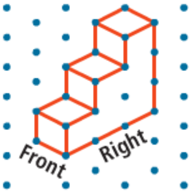

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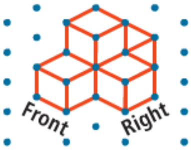

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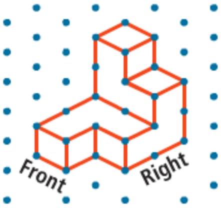
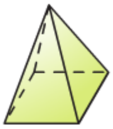


B Apply

20. **Multiple Representations** There are eight different nets for the solid shown at the right. Draw as many of them as you can. (*Hint*: Two nets are the same if you can rotate or flip one to match the other.)

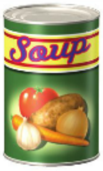


21. **a.** **Open-Ended** Make an isometric drawing of a structure that you can build using 8 cubes.

b. Make an orthographic drawing of this structure.

22. **Think About a Plan** Draw a net of the can at the right.

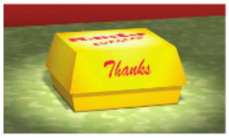
- What shape are the top and bottom of the can?
- If you uncurl the body of the can, what shape do you get?



23. **History** In 1525, German printmaker Albrecht Dürer first used the word *net* to describe a printed pattern that folds up into a three-dimensional shape. Why do you think he chose to use the word *net*?

Manufacturing **Match the package with its net.**

24.



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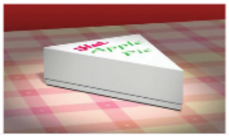

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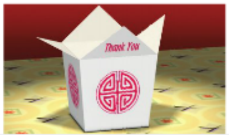
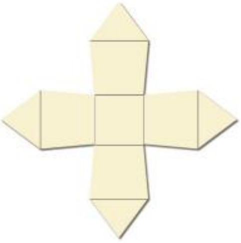


A.



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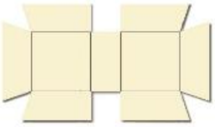

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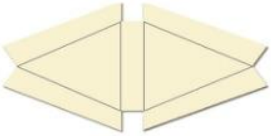
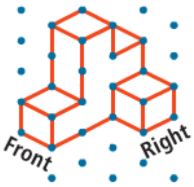
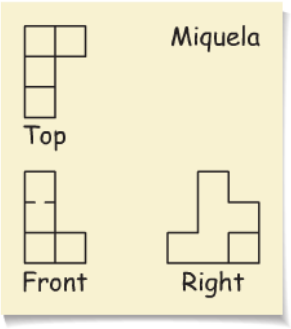
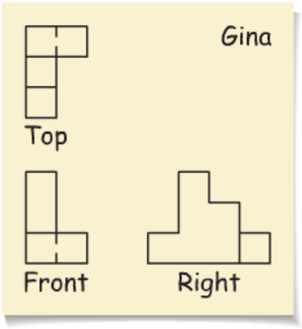




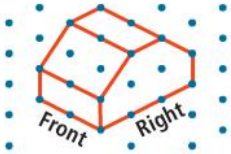
27. **Error Analysis** Miquela and Gina drew orthographic drawings for the cube structure at the right. Who is correct?





Make an orthographic drawing for each isometric drawing.

28.



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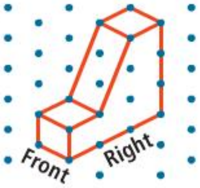

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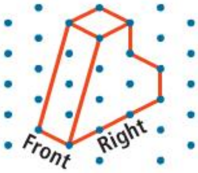
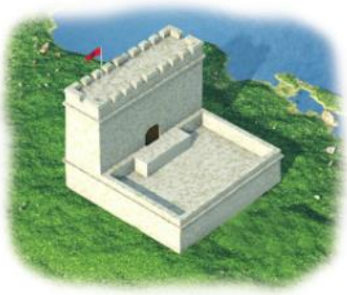


31. **Fort** Use the diagram of the fort at the right.

a. Make an isometric drawing of the fort.

b. Make an orthographic drawing of the fort.

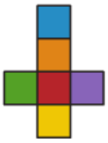


32. **Aerial Photography** Another perspective in aerial photography is the "bird's-eye view," which shows an object from directly overhead. What type of drawing that you have studied in this lesson is a bird's-eye view?

33. **Writing** Photographs of buildings are typically not taken from a bird's-eye view. Describe a situation in which you would want a photo showing a bird's-eye view.

Visualization **Think about how each net can be folded to form a cube. What is the color of the face that will be opposite the red face?**

34.



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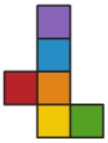

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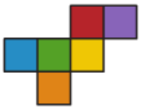

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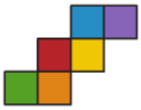
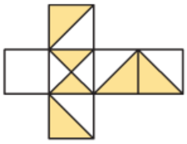


38. **Multiple Representations** There are 11 different nets for a cube. Four of them are shown above.

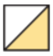
a. Draw the other seven nets.

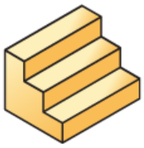
b. **Writing** Suppose you want to make 100 cubes for an art project. Which of the 11 nets would you use? Explain why.

Challenge



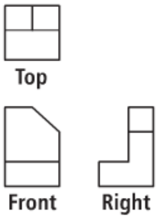
39. The net at the right folds into a cube. Sketch the cube so that its front face is shaded as shown below.





40. **Architecture** What does the net of the staircase shown look like? Draw the net. (*Hint*: Visualize stretching the stairs out flat.)

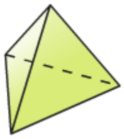
41. A hexomino is a two-dimensional figure formed with six squares. Each square shares at least one side with another square. The 11 nets of a cube that you found in Exercise 38 are hexominoes. Draw as many of the remaining 24 hexominoes as you can.



42. **Visualization** Use the orthographic drawing at the right.
- **a.** Make an isometric drawing of the structure.
- **b.** Make an isometric drawing of the structure from part (a) after it has been turned on its base 90° counterclockwise.
- **c.** Make an orthographic drawing of the structure from part (b).
- **d.** Turn the structure from part (a) 180°. Repeat parts (b) and (c).

Standardized Test Prep

SAT/ACT



43. How many possible nets does the solid at the right have?

(A) 1 (B) 2 (C) 3 (D) 4

44. Solve $10a - 5b = 25$ for b.

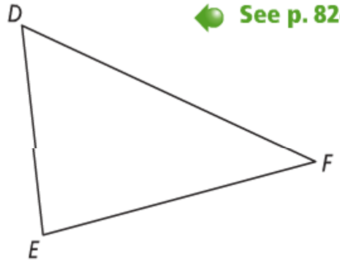
(F) $b = 10a + 25$ (G) $b = 10a - 25$ (H) $b = 2a + 5$ (I) $b = 2a - 5$

Short Response

45. Graph the equation $x + 2y = -3$. Label the x- and y-intercepts.

Mixed Review

For Exercises 46 and 47, use the diagram at the right.



See p. 824.

46. Measure DE and EF to the nearest millimeter.

47. Measure each angle to the nearest degree.

48. Draw a triangle that has sides of length 6 cm and 5 cm with a 90° angle between those two sides.

Get Ready! **To prepare for Lesson 1-2, do Exercises 49–51.**

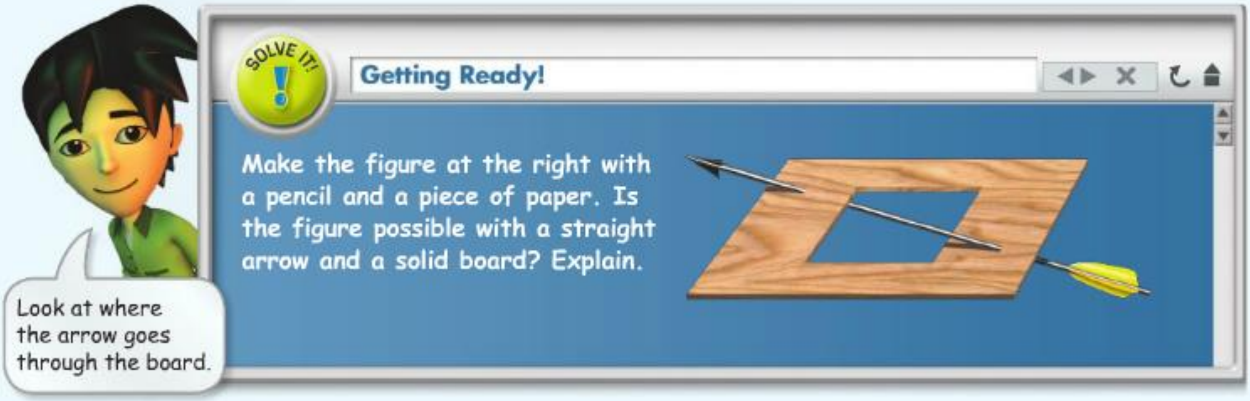
Coordinate Geometry **Graph the points on the coordinate plane.** **See p. 833.**

49. (0, 0), (2, 2), (0, 3) **50.** (1, 2), (−4, 3), (−5, 0) **51.** (−4, −5), (0, −1), (3, −2)

1-2

Points, Lines, and Planes

Objective To understand basic terms and postulates of geometry


SOLVE IT!
Getting Ready!
Look at where the arrow goes through the board.

Make the figure at the right with a pencil and a piece of paper. Is the figure possible with a straight arrow and a solid board? Explain.

In this lesson, you will learn basic geometric facts to help you justify your answer to the Solve It.

Lesson Vocabulary

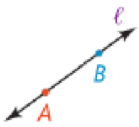
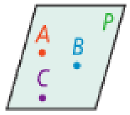
- point
- line
- plane
- collinear points
- coplanar
- space
- segment
- ray
- opposite rays
- postulate
- axiom
- intersection

Essential Understanding Geometry is a mathematical system built on accepted facts, basic terms, and definitions.

In geometry, some words such as *point*, *line*, and *plane* are undefined. Undefined terms are the basic ideas that you can use to build the definitions of all other figures in geometry. Although you cannot define undefined terms, it is important to have a general description of their meanings.

take note

Key Concept Undefined Terms

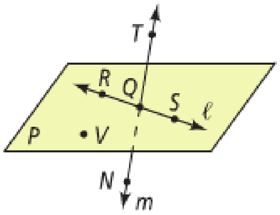
Term Description	How to Name It	Diagram
A **point** indicates a location and has no size.	You can represent a point by a dot and name it by a capital letter, such as *A*.	
A **line** is represented by a straight path that extends in two opposite directions without end and has no thickness. A line contains infinitely many points.	You can name a line by any two points on the line, such as $\overleftrightarrow{AB}$ (read "line *AB*") or $\overleftrightarrow{BA}$, or by a single lowercase letter, such as line ℓ.	
A **plane** is represented by a flat surface that extends without end and has no thickness. A plane contains infinitely many lines.	You can name a plane by a capital letter, such as plane *P*, or by at least three points in the plane that do not all lie on the same line, such as plane *ABC*.	

Points that lie on the same line are **collinear points.** Points and lines that lie in the same plane are **coplanar.** All the points of a line are coplanar.

Think

Why can figures have more than one name? Lines and planes are made up of many points. You can choose any two points on a line and any three or more noncollinear points in a plane for the name.

Problem 1 Naming Points, Lines and Planes



A What are two other ways to name $\overleftrightarrow{QT}$?

Two other ways to name $\overleftrightarrow{QT}$ are $\overleftrightarrow{TQ}$ and line *m*.

B What are two other ways to name plane *P*?

Two other ways to name plane *P* are plane *RQV* and plane *RSV*.

C What are the names of three collinear points? What are the names of four coplanar points?

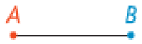
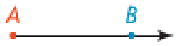
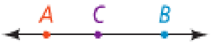
Points *R*, *Q*, and *S* are collinear. Points *R*, *Q*, *S*, and *V* are coplanar.

Got It? **1. a.** What are two other ways to name $\overleftrightarrow{RS}$?
b. What are two more ways to name plane *P*?
c. What are the names of three other collinear points?
d. What are two points that are *not* coplanar with points *R*, *S*, and *V*?

The terms *point*, *line*, and *plane* are not defined because their definitions would require terms that also need defining. You can, however, use undefined terms to define other terms. A geometric figure is a set of points. **Space** is the set of all points in three dimensions. Similarly, the definitions for *segment* and *ray* are based on the definitions of points and lines.

take note

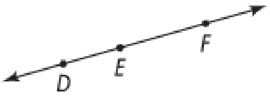
Key Concept Defined Terms

Definition	How to Name It	Diagram
A **segment** is part of a line that consists of two endpoints and all points between them.	You can name a segment by its two endpoints, such as $\overline{AB}$ (read "segment *AB*") or $\overline{BA}$.	
A **ray** is part of a line that consists of one endpoint and all the points of the line on one side of the endpoint.	You can name a ray by its endpoint and another point on the ray, such as $\overrightarrow{AB}$ (read "ray *AB*"). The order of points indicates the ray's direction.	
Opposite rays are two rays that share the same endpoint and form a line.	You can name opposite rays by their shared endpoint and any other point on each ray, such as $\overrightarrow{CA}$ and $\overrightarrow{CB}$.	

Plan

How do you make sure you name all the rays?
Each point on the line is an endpoint for a ray. At each point, follow the line both left and right to see if you can find a second point to name the ray.

Problem 2 Naming Segments and Rays



A **What are the names of the segments in the figure at the right?**

The three segments are $\overline{DE}$ or $\overline{ED}$, $\overline{EF}$ or $\overline{FE}$, and $\overline{DF}$ or $\overline{FD}$.

B **What are the names of the rays in the figure?**

The four rays are $\overrightarrow{DE}$ or $\overrightarrow{DF}$, $\overrightarrow{ED}$, $\overrightarrow{EF}$, and $\overrightarrow{FD}$ or $\overrightarrow{FE}$.

C **Which of the rays in part (B) are opposite rays?**

The opposite rays are $\overrightarrow{ED}$ and $\overrightarrow{EF}$.

Got It? **2.** **Reasoning** $\overrightarrow{EF}$ and $\overrightarrow{FE}$ form a line. Are they opposite rays? Explain.

A **postulate** or **axiom** is an accepted statement of fact. Postulates, like undefined terms, are basic building blocks of the logical system in geometry. You will use logical reasoning to prove general concepts in this book.

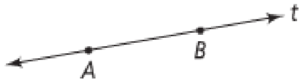
You have used some of the following geometry postulates in algebra. For example, you used Postulate 1-1 when you graphed equations such as $y = 2x + 8$. You graphed two points and drew the line through the points.

take note

Postulate 1-1

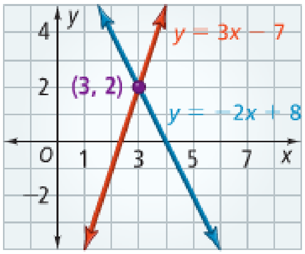
Through any two points there is exactly one line.

Line t passes through points A and B. Line t is the only line that passes through both points.



When you have two or more geometric figures, their **intersection** is the set of points the figures have in common.

In algebra, one way to solve a system of two equations is to graph them. The graphs of the two lines $y = -2x + 8$ and $y = 3x - 7$ intersect in a single point (3, 2). So the solution is (3, 2). This illustrates Postulate 1-2.

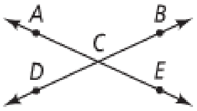


take note

Postulate 1-2

If two distinct lines intersect, then they intersect in exactly one point.

$\overleftrightarrow{AE}$ and $\overleftrightarrow{DB}$ intersect in point C.



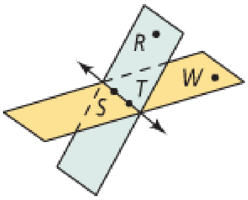
There is a similar postulate about the intersection of planes.

take note

Postulate 1-3

If two distinct planes intersect, then they intersect in exactly one line.

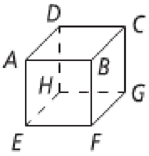
Plane *RST* and plane *WST* intersect in $\overleftrightarrow{ST}$.



When you know two points that two planes have in common, Postulates 1-1 and 1-3 tell you that the line through those points is the intersection of the planes.

Problem 3 Finding the Intersection of Two Planes

Each surface of the box at the right represents part of a plane. What is the intersection of plane *ADC* and plane *BFG*?



Know

Plane *ADC* and plane *BFG*

Need

The intersection of the two planes

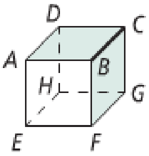
Plan

Find the points that the planes have in common.

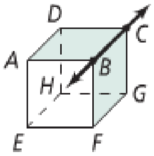
Think

Is the intersection a segment?

No. The intersection of the sides of the box is a segment, but planes continue without end. The intersection is a line.



Focus on plane *ADC* and plane *BFG* to see where they intersect.



You can see that both planes contain point *B* and point *C*.

The planes intersect in $\overleftrightarrow{BC}$.

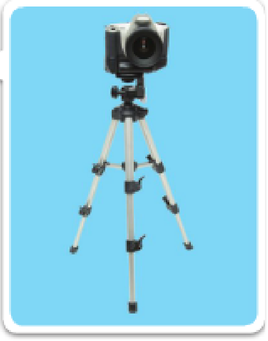
Got It? **3. a.** What are the names of two planes that intersect in $\overleftrightarrow{BF}$?

b. **Reasoning** Why do you only need to find two common points to name the intersection of two distinct planes?

When you name a plane from a figure like the box in Problem 3, list the corner points in consecutive order. For example, plane *ADCB* and plane *ABCD* are also names for the plane on the top of the box. Plane *ACBD* is not.

Photographers use three-legged tripods to make sure that a camera is steady. The feet of the tripod all touch the floor at the same time. You can think of the feet as points and the floor as a plane. As long as the feet do not all lie in one line, they will lie in exactly one plane.

This illustrates Postulate 1-4.

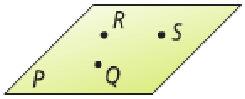


take note

Postulate 1-4

Through any three noncollinear points there is exactly one plane.

Points *Q*, *R*, and *S* are noncollinear. Plane *P* is the only plane that contains them.

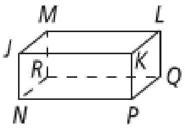


Problem 4 Using Postulate 1-4

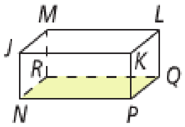
Plan

How can you find the plane?
Try to draw all the lines that contain two of the three given points. You will begin to see a plane form.

Use the figure at the right.

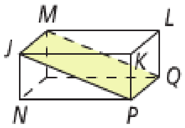


A **What plane contains points *N*, *P*, and *Q*? Shade the plane.**



The plane on the bottom of the figure contains points *N*, *P*, and *Q*.

B **What plane contains points *J*, *M*, and *Q*? Shade the plane.**



The plane that passes at a slant through the figure contains points *J*, *M*, and *Q*.

Got It? **4. a.** What plane contains points *L*, *M*, and *N*? Copy the figure in Problem 4 and shade the plane.

b. **Reasoning** What is the name of a line that is coplanar with $\overleftrightarrow{JK}$ and $\overleftrightarrow{KL}$?

Lesson Check

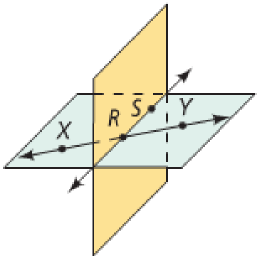
Do you know HOW?

Use the figure at the right.

1. What are two other names for $\overleftrightarrow{XY}$?

2. What are the opposite rays?

3. What is the intersection of the two planes?



Do you UNDERSTAND?

4. **Vocabulary** A segment has endpoints R and S. What are two names for the segment?

5. Are $\overrightarrow{AB}$ and $\overrightarrow{BA}$ the same ray? Explain.

6. **Reasoning** Why do you use two arrowheads when drawing or naming a line such as $\overleftrightarrow{EF}$?

7. **Compare and Contrast** How is naming a ray similar to naming a line? How is it different?

Practice and Problem-Solving Exercises

A Practice

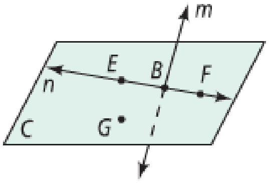
Use the figure at the right for Exercises 8–11. **See Problem 1.**

8. What are two other ways to name $\overleftrightarrow{EF}$?

9. What are two other ways to name plane C?

10. Name three collinear points.

11. Name four coplanar points.

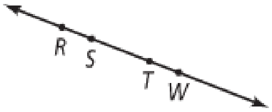


Use the figure at the right for Exercises 12–14. **See Problem 2.**

12. Name the segments in the figure.

13. Name the rays in the figure.

14. **a.** Name the pair of opposite rays with endpoint T.
b. Name another pair of opposite rays.



Use the figure at the right for Exercises 15–26.

Name the intersection of each pair of planes. **See Problem 3.**

15. planes QRS and RSW

16. planes UXV and WVS

17. planes XWV and UVR

18. planes TXW and TQU

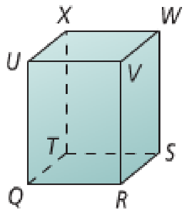
Name two planes that intersect in the given line.

19. $\overleftrightarrow{QU}$

20. $\overleftrightarrow{TS}$

21. $\overleftrightarrow{XT}$

22. $\overleftrightarrow{VW}$



Copy the figure. Shade the plane that contains the given points. **See Problem 4.**

23. R, V, W

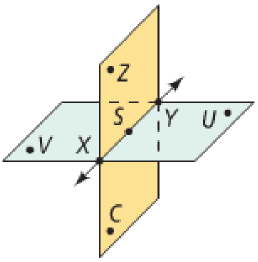
24. U, V, W

25. U, X, S

26. T, U, V

B Apply

Postulate 1-4 states that any three noncollinear points lie in exactly one plane. Find the plane that contains the first three points listed. Then determine whether the fourth point is in that plane. Write *coplanar* or *noncoplanar* to describe the points.



27. Z, S, Y, C

28. S, U, V, Y

29. X, Y, Z, U

30. X, S, V, U

31. X, Z, S, V

32. S, V, C, Y

If possible, draw a figure to fit each description. Otherwise, write *not possible.*

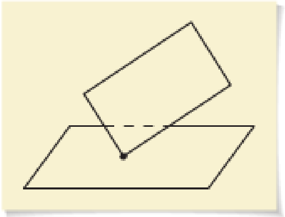
33. four points that are collinear

34. two points that are noncollinear

35. three points that are noncollinear

36. three points that are noncoplanar

37. **Open-Ended** Draw a figure with points B, C, D, E, F, and G that shows $\overleftrightarrow{CD}$, $\overleftrightarrow{BG}$, and $\overleftrightarrow{EF}$, with one of the points on all three lines.



38. **Think About a Plan** Your friend drew the diagram at the right to prove to you that two planes can intersect in exactly one point. Describe your friend's error.

- How do you describe a plane?
- What does it mean for two planes to intersect each other?
- Can you define an endpoint of a plane?

39. **Reasoning** If one ray contains another ray, are they the same ray? Explain.

For Exercises 40–45, determine whether each statement is *always, sometimes,* or *never* true.

40. $\overleftrightarrow{TQ}$ and $\overleftrightarrow{QT}$ are the same line.

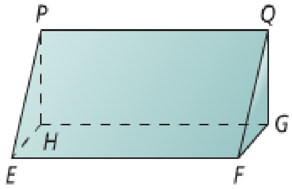
41. $\overrightarrow{JK}$ and $\overrightarrow{JL}$ are the same ray.

42. Intersecting lines are coplanar.

43. Four points are coplanar.

44. A plane containing two points of a line contains the entire line.

45. Two distinct lines intersect in more than one point.



46. Use the diagram at the right. How many planes contain each line and point?

a. $\overleftrightarrow{EF}$ and point G

b. $\overleftrightarrow{PH}$ and point E

c. $\overleftrightarrow{FG}$ and point P

d. $\overleftrightarrow{EP}$ and point G

e. **Reasoning** What do you think is true of a line and a point not on the line? Explain. (*Hint*: Use two of the postulates you learned in this lesson.)

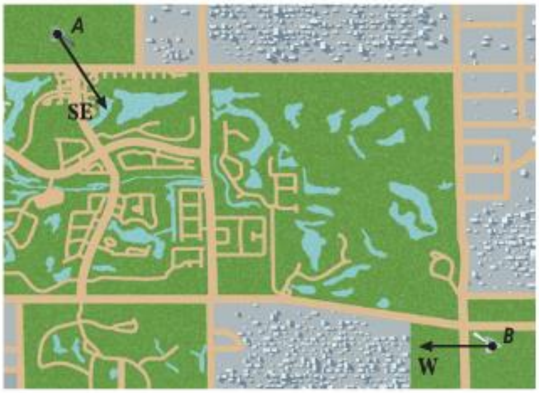
In Exercises 47–49, sketch a figure for the given information. Then state the postulate that your figure illustrates.

47. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ intersect in point C.

48. The noncollinear points A, B, and C are all contained in plane N.

49. Planes LNP and MVK intersect in $\overleftrightarrow{NM}$.

50. **Telecommunications** A cell phone tower at point A receives a cell phone signal from the southeast. A cell phone tower at point B receives a signal from the same cell phone from due west. Trace the diagram at the right and find the location of the cell phone. Describe how Postulates 1-1 and 1-2 help you locate the phone.



51. **Estimation** You can represent the hands on a clock at 6:00 as opposite rays. Estimate the other 11 times on a clock that you can represent as opposite rays.

52. **Open-Ended** What are some basic words in English that are difficult to define?

Coordinate Geometry **Graph the points and state whether they are collinear.**

53. (1, 1), (4, 4), (−3, −3)

54. (2, 4), (4, 6), (0, 2)

55. (0, 0), (−5, 1), (6, −2)

56. (0, 0), (8, 10), (4, 6)

57. (0, 0), (0, 3), (0, −10)

58. (−2, −6), (1, −2), (4, 1)

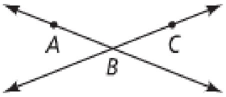
Challenge

59. How many planes contain the same three collinear points? Explain.

60. How many planes contain a given line? Explain.

61. **a.** **Writing** Suppose two points are in plane P. Explain why the line containing the points is also in plane P.

b. **Reasoning** Suppose two lines intersect. How many planes do you think contain both lines? Use the diagram at the right and your answer to part (a) to explain your answer.

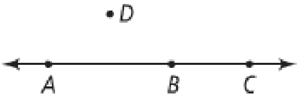


Probability **Suppose you pick points at random from A, B, C, and D shown below. Find the probability that the number of points given meets the condition stated.**

62. 2 points, collinear

63. 3 points, collinear

64. 3 points, coplanar



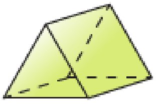
Standardized Test Prep

SAT/ACT

65. Which geometric term is undefined?

Ⓐ segment
Ⓒ ray
Ⓑ collinear
Ⓓ plane

66. Which diagram is a net of the figure shown at the right?



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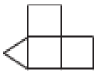

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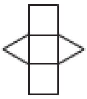

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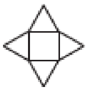

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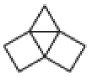


67. You want to cut a block of cheese into four pieces. What is the least number of cuts you need to make?

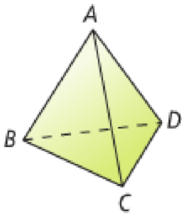
Ⓐ 2
Ⓑ 3
Ⓒ 4
Ⓓ 5

Short Response

68. The figure at the right is called a tetrahedron.

a. Name all the planes that form the surfaces of the tetrahedron.

b. Name all the lines that intersect at *D*.



Mixed Review

Make an orthographic drawing for each figure. Assume there are no hidden cubes. **See Lesson 1-1.**

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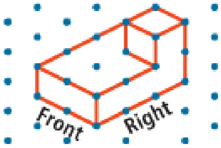
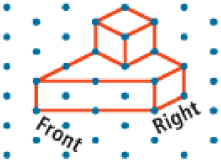
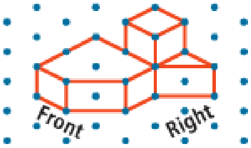


70.


71.


Simplify each ratio. **See p. 831.**

72. 30 to 12

73. $\frac{15x}{35x}$

74. $\frac{n^2 + n}{4n}$

Get Ready! **To prepare for Lesson 1-3, do Exercises 75–80.**

Simplify each absolute value expression. **See p. 832.**

75. $|-6|$

76. $|3.5|$

77. $|7 - 10|$

Algebra Solve each equation. **See p. 834.**

78. $x + 2x - 6 = 6$

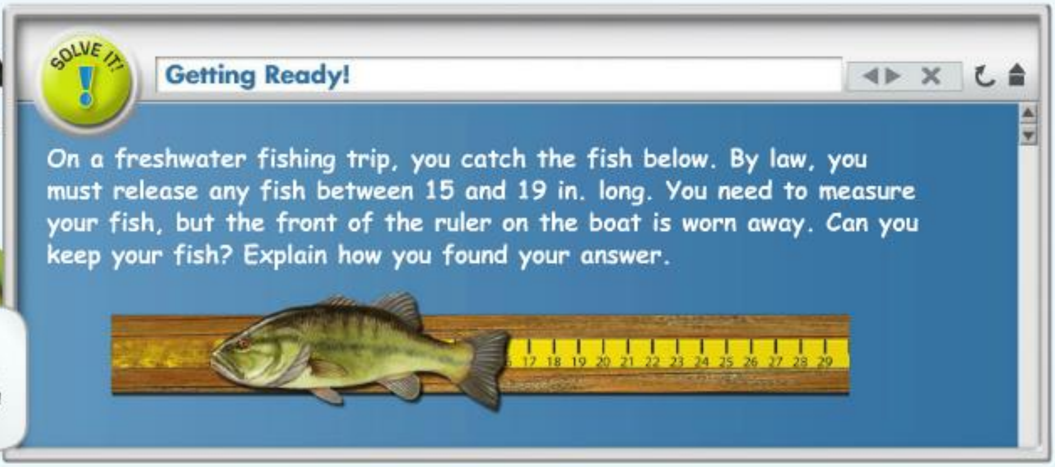
79. $3x + 9 + 5x = 81$

80. $w - 2 = -4 + 7w$

1-3 Measuring Segments

Objective To find and compare lengths of segments

The fish isn't at zero, but you can still find how long it is.



On a freshwater fishing trip, you catch the fish below. By law, you must release any fish between 15 and 19 in. long. You need to measure your fish, but the front of the ruler on the boat is worn away. Can you keep your fish? Explain how you found your answer.

Lesson Vocabulary
- coordinate
- distance
- congruent segments
- midpoint
- segment bisector

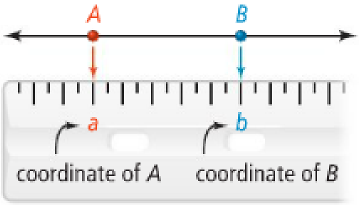
In the Solve It, you measured the length of an object indirectly.

Essential Understanding You can use number operations to find and compare the lengths of segments.

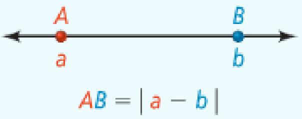
take note

Postulate 1-5 Ruler Postulate

Every point on a line can be paired with a real number. This makes a one-to-one correspondence between the points on the line and the real numbers. The real number that corresponds to a point is called the **coordinate** of the point.



The Ruler Postulate allows you to measure lengths of segments using a given unit and to find distances between points on a number line. Consider $\overleftrightarrow{AB}$ at the right. The **distance** between points A and B is the absolute value of the difference of their coordinates, or $|a - b|$. This value is also AB, or the length of $\overline{AB}$.

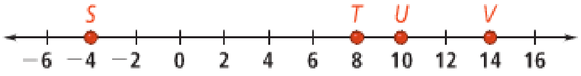


Problem 1 Measuring Segment Lengths

Think

What are you trying to find?
ST represents the length of $\overline{ST}$, so you are trying to find the distance between points S and T.

What is ST?



The coordinate of S is -4.

Ruler Postulate

The coordinate of T is 8.

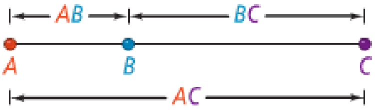
$$\begin{aligned} ST &= \left|-4 - 8\right| && \text{Definition of distance} \\ &= \left|-12\right| && \text{Subtract.} \\ &= 12 && \text{Find the absolute value.} \end{aligned}$$

Got It? **1.** What are UV and SV on the number line above?

take note

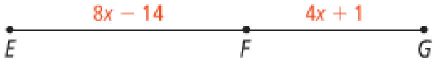
Postulate 1-6 Segment Addition Postulate

If three points A, B, and C are collinear and B is between A and C, then $AB + BC = AC$.



Problem 2 Using the Segment Addition Postulate

Algebra If $EG = 59$, what are EF and FG?



Algebra If $EG = 59$, what are EF and FG?

Know
$EG = 59$
$EF = 8x - 14$
$FG = 4x + 1$

Need
EF and FG

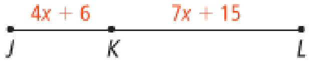
Plan
Use the Segment Addition Postulate to write an equation.

$$\begin{aligned} EF + FG &= EG && \text{Segment Addition Postulate} \\ (8x - 14) + (4x + 1) &= 59 && \text{Substitute.} \\ 12x - 13 &= 59 && \text{Combine like terms.} \\ 12x &= 72 && \text{Add 13 to each side.} \\ x &= 6 && \text{Divide each side by 12.} \end{aligned}$$

Use the value of x to find EF and FG.

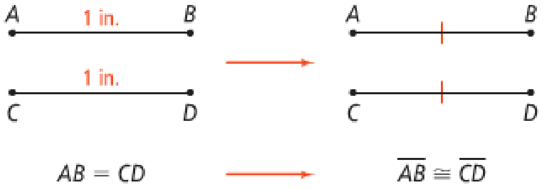
$$\begin{aligned} EF &= 8x - 14 = 8(6) - 14 = 48 - 14 = 34 \\ FG &= 4x + 1 = 4(6) + 1 = 24 + 1 = 25 \end{aligned} \quad \text{Substitute 6 for } x.$$

Got It? **2.** In the diagram, $JL = 120$. What are JK and KL?



When numerical expressions have the same value, you say that they are equal (=). Similarly, if two segments have the same length, then the segments are **congruent (≅) segments.**

This means that if $AB = CD$, then $\overline{AB} \cong \overline{CD}$. You can also say that if $\overline{AB} \cong \overline{CD}$, then $AB = CD$.



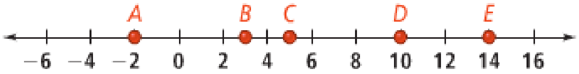
As illustrated above, you can mark segments alike to show that they are congruent. If there is more than one set of congruent segments, you can indicate each set with the same number of marks.

Problem 3 Comparing Segment Lengths

Plan

How do you know if segments are congruent?
Congruent segments have the same length. So find and compare the lengths of $\overline{AC}$ and $\overline{BD}$.

Are $\overline{AC}$ and $\overline{BD}$ congruent?



$AC = |-2 - 5| = |-7| = 7$

Definition of distance

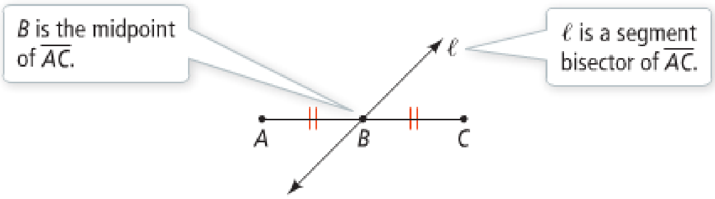
$BD = |3 - 10| = |-7| = 7$

Yes. $AC = BD$, so $\overline{AC} \cong \overline{BD}$.

Got It? **3. a.** Use the diagram above. Is $\overline{AB}$ congruent to $\overline{DE}$?

b. Reasoning To find AC in Problem 3, suppose you subtract −2 from 5. Do you get the same result? Why?

The **midpoint** of a segment is a point that divides the segment into two congruent segments. A point, line, ray, or other segment that intersects a segment at its midpoint is said to *bisect* the segment. That point, line, ray, or segment is called a **segment bisector.**

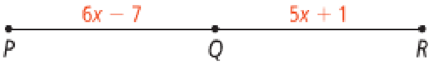

B is the midpoint of $\overline{AC}$.
ℓ is a segment bisector of $\overline{AC}$.
ℓ
A
B
C

Plan

How can you use algebra to solve the problem?
The lengths of the congruent segments are given as algebraic expressions. You can set the expressions equal to each other.

Problem 4 Using the Midpoint

Algebra Q is the midpoint of $\overline{PR}$. What are PQ, QR, and PR?



Step 1 Find x.

$PQ = QR$ Definition of midpoint

$6x - 7 = 5x + 1$ Substitute.

$x - 7 = 1$ Subtract 5x from each side.

$x = 8$ Add 7 to each side.

Step 2 Find PQ and QR.

$PQ = 6x - 7$

$= 6(8) - 7$ Substitute 8 for x.

$= 41$ Simplify.

$QR = 5x + 1$

$= 5(8) + 1$

$= 41$

Step 3 Find PR.

$PR = PQ + QR$ Segment Addition Postulate

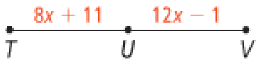
$= 41 + 41$ Substitute.

$= 82$ Simplify.

PQ and QR are both 41. PR is 82.

Got It? **4. a. Reasoning** Is it necessary to substitute 8 for x in the expression for QR in order to find QR? Explain.

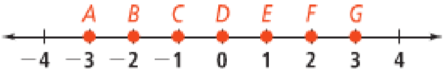
b. U is the midpoint of $\overline{TV}$. What are TU, UV, and TV?



Lesson Check

Do you know HOW?

Name each of the following.



1. The point on $\overrightarrow{DA}$ that is 2 units from D

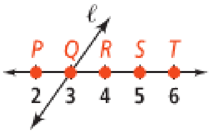
2. Two points that are 3 units from D

3. The coordinate of the midpoint of $\overline{AG}$

4. A segment congruent to $\overline{AC}$

Do you UNDERSTAND?

5. Vocabulary Name two segment bisectors of $\overline{PR}$.



6. Compare and Contrast Describe the difference between saying that two segments are *congruent* and saying that two segments have *equal length*. When would you use each phrase?

7. Error Analysis You and your friend live 5 mi apart. He says that it is 5 mi from his house to your house and −5 mi from your house to his house. What is the error in his argument?

Practice and Problem-Solving Exercises

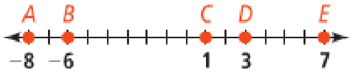
A Practice

Find the length of each segment.

See Problem 1.

8. $\overline{AB}$ **9.** $\overline{BD}$

10. $\overline{AD}$ **11.** $\overline{CE}$

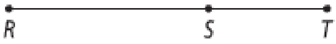


Use the number line at the right for Exercises 12–14.

See Problem 2.

12. If $RS = 15$ and $ST = 9$, then $RT =$ ■.

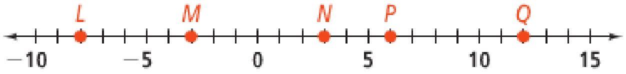
13. If $ST = 15$ and $RT = 40$, then $RS =$ ■.



14. **Algebra** $RS = 8y + 4$, $ST = 4y + 8$, and $RT = 15y - 9$.
 a. What is the value of y?
 b. Find RS, ST, and RT.

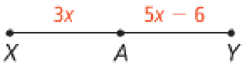
Use the number line below for Exercises 15–18. Tell whether the segments are congruent.

See Problem 3.



15. $\overline{LN}$ and $\overline{MQ}$ **16.** $\overline{MP}$ and $\overline{NQ}$ **17.** $\overline{MN}$ and $\overline{PQ}$ **18.** $\overline{LP}$ and $\overline{MQ}$

19. **Algebra** A is the midpoint of $\overline{XY}$.
 a. Find XA.
 b. Find AY and XY.



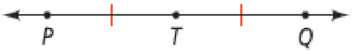
See Problem 4.

Algebra **For Exercises 20–22, use the figure below. Find the value of PT.**

20. $PT = 5x + 3$ and $TQ = 7x - 9$

21. $PT = 4x - 6$ and $TQ = 3x + 4$

22. $PT = 7x - 24$ and $TQ = 6x - 2$



B Apply

On a number line, the coordinates of X, Y, Z, and W are -7, -3, 1, and 5, respectively. Find the lengths of the two segments. Then tell whether they are congruent.

23. $\overline{XY}$ and $\overline{ZW}$ **24.** $\overline{ZX}$ and $\overline{WY}$ **25.** $\overline{YZ}$ and $\overline{XW}$

Suppose the coordinate of A is 0, $AR = 5$, and $AT = 7$. What are the possible coordinates of the midpoint of the given segment?

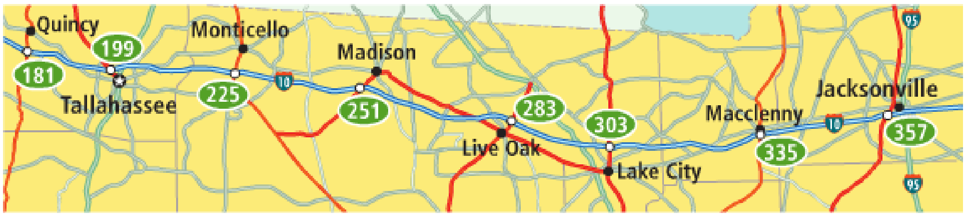
26. $\overline{AR}$ **27.** $\overline{AT}$ **28.** $\overline{RT}$

29. Suppose point E has a coordinate of 3 and $EG = 5$. What are the possible coordinates of point G?

Visualization **Without using your ruler, sketch a segment with the given length. Use your ruler to see how well your sketch approximates the length provided.**

30. 3 cm **31.** 3 in. **32.** 6 in. **33.** 10 cm **34.** 65 mm

35. **Think About a Plan** The numbers labeled on the map of Florida are mile markers. Assume that Route 10 between Quincy and Jacksonville is straight.

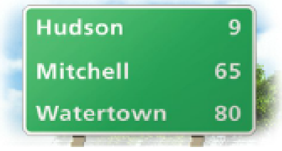


Suppose you drive at an average speed of 55 mi/h. How long will it take to get from Live Oak to Jacksonville?

- How can you use mile markers to find distances between points?
- How do average speed, distance, and time all relate to each other?

36. **Travel** Use the map above. Suppose you drive at an average speed of 58 mi/h. How long will it take to get from Macclenny to Tallahassee?

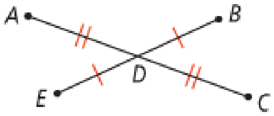
Error Analysis **Use the highway sign for Exercises 37 and 38.**



37. A driver reads the highway sign and says, "It's 145 miles from Mitchell to Watertown." What error did the driver make? Explain.

38. Your friend reads the highway sign and says, "It's 71 miles to Watertown." Is your friend correct? Explain.

Algebra **Use the diagram at the right for Exercises 39 and 40.**



39. If $AD = 12$ and $AC = 4y - 36$, find the value of y. Then find AC and DC.

40. If $ED = x + 4$ and $DB = 3x - 8$, find ED, DB, and EB.

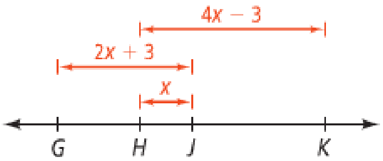
41. **Writing** Suppose you know PQ and QR. Can you use the Segment Addition Postulate to find PR? Explain.

Challenge

42. C is the midpoint of $\overline{AB}$, D is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{ED}$, G is the midpoint of $\overline{EF}$, and H is the midpoint of $\overline{DB}$. If $DC = 16$, what is GH?

43. a. **Algebra** Use the diagram at the right. What algebraic expression represents GK?

b. If $GK = 30$, what are GH and JK?



Standardized Test Prep

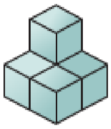
SAT/ACT

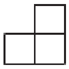
44. Points X, Y, and Z are collinear and Y is between X and Z. Which statement must be true?

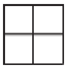
Ⓐ $XY = YZ$ Ⓒ $XY + XZ = YZ$

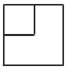
Ⓑ $XZ - XY = YZ$ Ⓓ $XZ = XY - YZ$

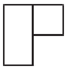
45. Which is the top view of an orthographic drawing of the figure at the right?



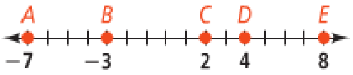
Ⓕ 

Ⓗ 

Ⓖ 

Ⓘ 

46. Which statement is true based on the diagram?

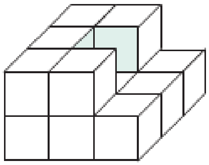


Ⓐ $\overline{BC} \cong \overline{CE}$ Ⓒ $AC + BD = AD$

Ⓑ $BD < CD$ Ⓓ $AC + CD = AD$

Extended Response

47. Make an orthographic drawing of the structure at the right.



Mixed Review

Complete each statement with *always, sometimes,* or *never* to make a true statement.

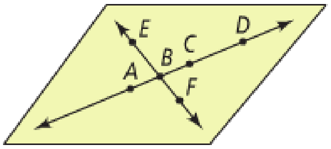
See Lesson 1-2.

48. Opposite rays $\underline{\;?\;}$ form a line.

49. Three distinct points are $\underline{\;?\;}$ coplanar.

50. If two distinct planes intersect, then their intersection is $\underline{\;?\;}$ a plane.

51. The intersection of two distinct planes is $\underline{\;?\;}$ a line.



52. Can you conclude the information stated from the given diagram?

a. A, B, and D are collinear. **b.** $\overline{AB} \cong \overline{BC}$

c. $\overrightarrow{BC}$ contains A. **d.** E, F, and B are coplanar.

Get Ready! **To prepare for Lesson 1-4, do Exercises 53–56.**

Algebra Solve the equation.

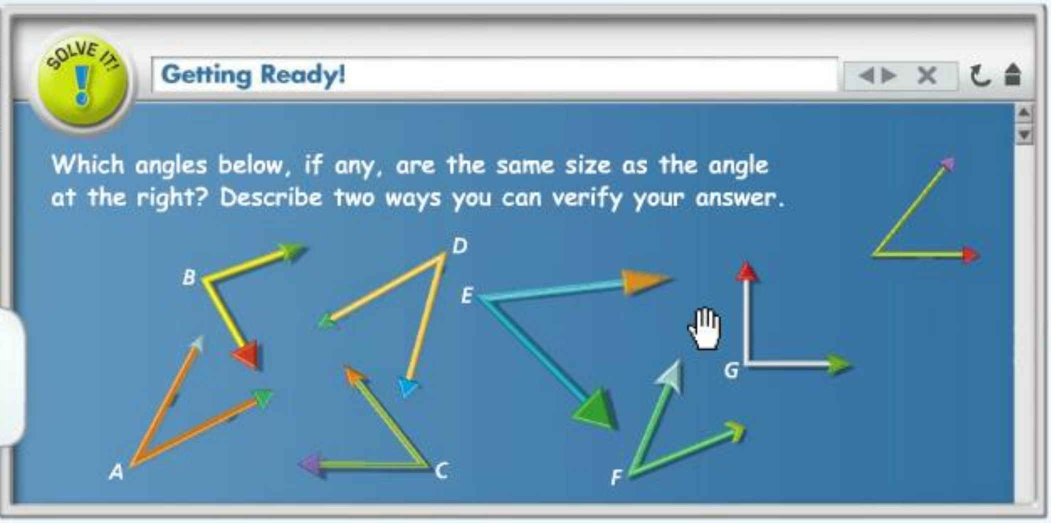
See p. 834.

53. $2x + 7 = 35$ **54.** $3y = 19.5$ **55.** $4z + 21 = 9$ **56.** $5t - 16 = 48$

1-4 Measuring Angles

Objective To find and compare the measures of angles

Think about what makes one angle greater than another.



Which angles below, if any, are the same size as the angle at the right? Describe two ways you can verify your answer.

Lesson Vocabulary

- angle
- sides of an angle
- vertex of an angle
- measure of an angle
- acute angle
- right angle
- obtuse angle
- straight angle
- congruent angles

In this lesson, you will learn to describe and measure angles like the ones in the Solve It.

Essential Understanding You can use number operations to find and compare the measures of angles.

take note

Key Concept Angle

Definition

An **angle** is formed by two rays with the same endpoint.

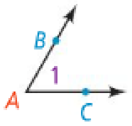
The rays are the **sides** of the angle. The endpoint is the **vertex** of the angle.

How to Name It

You can name an angle by

- its vertex, $\angle A$
- a point on each ray and the vertex, $\angle BAC$ or $\angle CAB$
- a number, $\angle 1$

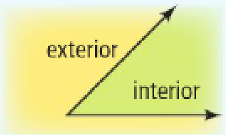
Diagram



The sides of the angle are $\overrightarrow{AB}$ and $\overrightarrow{AC}$. The vertex is A.

When you name angles using three points, the vertex must go in the middle.

The *interior* of an angle is the region containing all of the points between the two sides of the angle. The *exterior* of an angle is the region containing all of the points outside of the angle.



Think

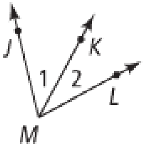
What rays form ∠1?

$\overrightarrow{MJ}$ and $\overrightarrow{MK}$ form ∠1.

Problem 1 Naming Angles

What are two other names for ∠1?

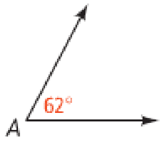
∠*JMK* and ∠*KMJ* are also names for ∠1.



Got It? **1. a.** What are two other names for ∠*KML*?

b. **Reasoning** Would it be correct to name any of the angles ∠*M*? Explain.

One way to measure the size of an angle is in degrees. To indicate the measure of an angle, write a lowercase *m* in front of the angle symbol. In the diagram, the measure of ∠*A* is 62. You write this as $m\angle A = 62$. In this book, you will work only with degree measures.

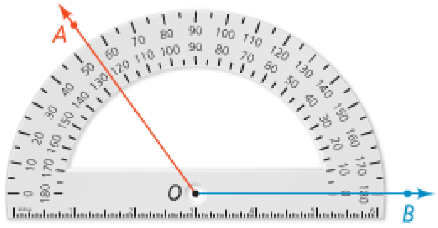


A circle has 360°, so 1 degree is $\frac{1}{360}$ of a circle. A protractor forms half a circle and measures angles from 0° to 180°.

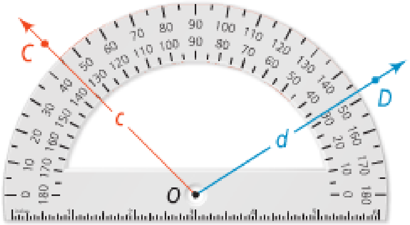
take note

Postulate 1-7 Protractor Postulate

Consider $\overrightarrow{OB}$ and a point *A* on one side of $\overrightarrow{OB}$. Every ray of the form $\overrightarrow{OA}$ can be paired one to one with a real number from 0 to 180.



The Protractor Postulate allows you to find the measure of an angle. Consider the diagram below. The **measure** of ∠*COD* is the absolute value of the difference of the real numbers paired with $\overrightarrow{OC}$ and $\overrightarrow{OD}$. That is, if $\overrightarrow{OC}$ corresponds with *c*, and $\overrightarrow{OD}$ corresponds with *d*, then $m\angle COD = |c - d|$.



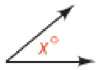
Notice that the Protractor Postulate and the calculation of an angle measure are very similar to the Ruler Postulate and the calculation of a segment length.

You can classify angles according to their measures.

take note

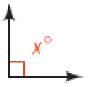
Key Concept Types of Angles

acute angle



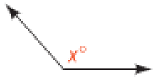
$0 < x < 90$

right angle



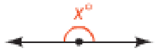
$x = 90$

obtuse angle



$90 < x < 180$

straight angle

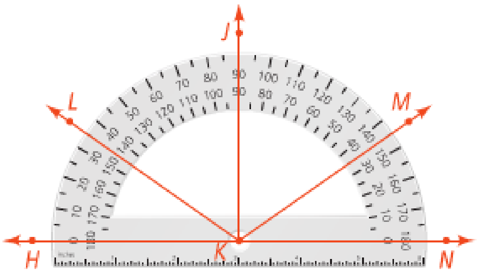


$x = 180$

The symbol ⌝ in the diagram above indicates a right angle.

Problem 2 Measuring and Classifying Angles

What are the measures of $\angle LKN$, $\angle JKL$, and $\angle JKN$? Classify each angle as *acute*, *right*, *obtuse*, or *straight*.



Think

Do the classifications make sense?
Yes. In each case, the classification agrees with what you see in the diagram.

Use the definition of the measure of an angle to calculate each measure.

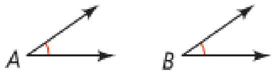
$m\angle LKN = |145 - 0| = 145$; $\angle LKN$ is obtuse.

$m\angle JKL = |90 - 145| = |-55| = 55$; $\angle JKL$ is acute.

$m\angle JKN = |90 - 0| = 90$; $\angle JKN$ is right.

Got It? **2.** What are the measures of $\angle LKH$, $\angle HKN$, and $\angle MKH$? Classify each angle as *acute*, *right*, *obtuse*, or *straight*.

Angles with the same measure are **congruent angles.** This means that if $m\angle A = m\angle B$, then $\angle A \cong \angle B$. You can also say that if $\angle A \cong \angle B$, then $m\angle A = m\angle B$.



$m\angle A = m\angle B$
$\angle A \cong \angle B$

You can mark angles with arcs to show that they are congruent. If there is more than one set of congruent angles, each set is marked with the same number of arcs.

Think

Look at the diagram. What do the angle marks tell you?

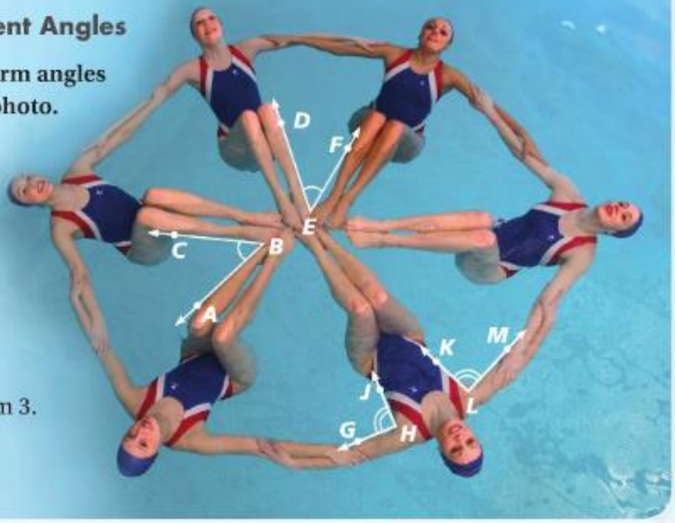
The angle marks tell you which angles are congruent.

Problem 3 Using Congruent Angles

Sports **Synchronized swimmers form angles with their bodies, as shown in the photo. If $m\angle GHJ = 90$, what is $m\angle KLM$?**

$\angle GHJ \cong \angle KLM$ because they both have two arcs.

So, $m\angle GHJ = m\angle KLM = 90$.



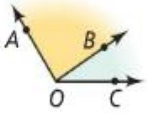
Got It? 3. Use the photo in Problem 3. If $m\angle ABC = 49$, what is $m\angle DEF$?

The Angle Addition Postulate is similar to the Segment Addition Postulate.

take note

Postulate 1-8 Angle Addition Postulate

If point B is in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$.



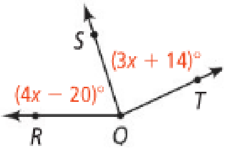
Plan

How can you use the expressions in the diagram?

The algebraic expressions represent the measures of the smaller angles, so they add up to the measure of the larger angle.

Problem 4 Using the Angle Addition Postulate

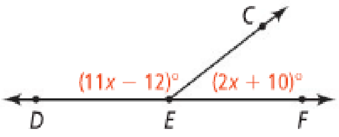
Algebra **If $m\angle RQT = 155$, what are $m\angle RQS$ and $m\angle TQS$?**



$$\begin{aligned} m\angle RQS + m\angle TQS &= m\angle RQT && \text{Angle Addition Postulate} \\ (4x - 20) + (3x + 14) &= 155 && \text{Substitute.} \\ 7x - 6 &= 155 && \text{Combine like terms.} \\ 7x &= 161 && \text{Add 6 to each side.} \\ x &= 23 && \text{Divide each side by 7.} \end{aligned}$$

$$\left.\begin{aligned} m\angle RQS &= 4x - 20 = 4(23) - 20 = 92 - 20 = 72 \\ m\angle TQS &= 3x + 14 = 3(23) + 14 = 69 + 14 = 83 \end{aligned}\right\} \text{Substitute 23 for } x.$$

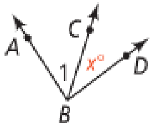
Got It? 4. $\angle DEF$ is a straight angle. What are $m\angle DEC$ and $m\angle CEF$?



Lesson Check

Do you know HOW?

Use the diagram for Exercises 1–3.



1. What are two other names for $\angle 1$?

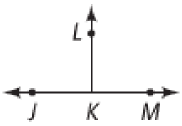
2. **Algebra** If $m\angle ABD = 85$, what is an expression to represent $m\angle ABC$?

3. Classify $\angle ABC$.

Do you UNDERSTAND?

4. **Vocabulary** How many sides can two congruent angles share? Explain.

5. **Error Analysis** Your classmate concludes from the diagram below that $\angle JKL \cong \angle LKM$. Is your classmate correct? Explain.



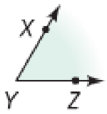
Practice and Problem-Solving Exercises

A Practice

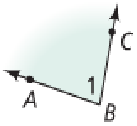
Name each shaded angle in three different ways.

See Problem 1.

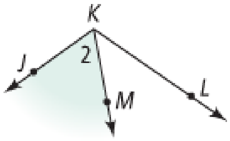
6.



7.



8.



Use the diagram below. Find the measure of each angle. Then classify the angle as *acute*, *right*, *obtuse*, or *straight*.

See Problem 2.

9. $\angle EAF$

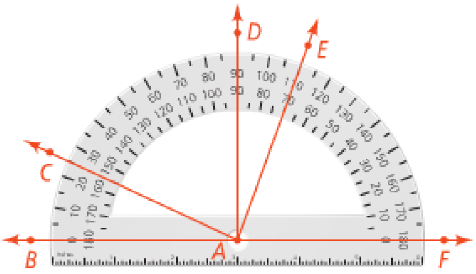
10. $\angle DAF$

11. $\angle BAE$

12. $\angle BAC$

13. $\angle CAE$

14. $\angle DAE$



Draw a figure that fits each description.

15. an obtuse angle, $\angle RST$

16. an acute angle, $\angle GHJ$

17. a straight angle, $\angle KLM$

Use the diagram below. Complete each statement.

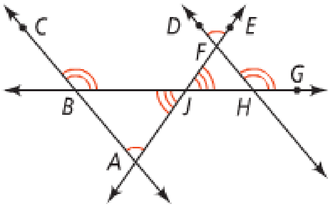
See Problem 3.

18. $\angle CBJ \cong$ ■

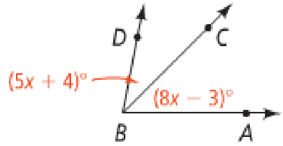
19. $\angle FJH \cong$ ■

20. If $m\angle EFD = 75$, then $m\angle JAB =$ ■.

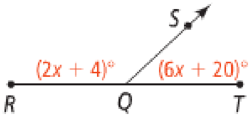
21. If $m\angle GHF = 130$, then $m\angle JBC =$ ■.



22. If $m\angle ABD = 79$, what are $m\angle ABC$ and $m\angle DBC$?



23. $\angle RQT$ is a straight angle. What are $m\angle RQS$ and $m\angle TQS$?

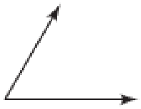


See Problem 4.

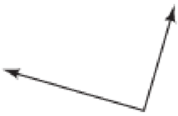
B Apply

Use a protractor. Measure and classify each angle.

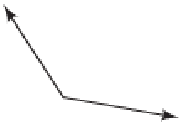
24.



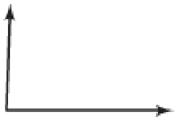
25.



26.

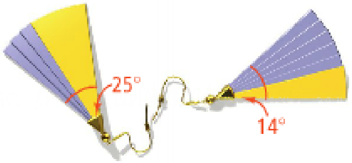


27.

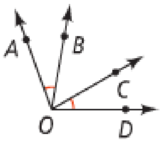


28. Think About a Plan A pair of earrings has blue wedges that are all the same size. One earring has a 25° yellow wedge. The other has a 14° yellow wedge. Find the angle measure of a blue wedge.

- How do the angle measures of the earrings relate?
- How can you use algebra to solve the problem?



Algebra **Use the diagram at the right for Exercises 29 and 30. Solve for *x*. Find the angle measures to check your work.**



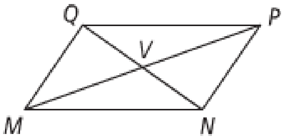
29. $m\angle AOB = 4x - 2$, $m\angle BOC = 5x + 10$, $m\angle COD = 2x + 14$

30. $m\angle AOB = 28$, $m\angle BOC = 3x - 2$, $m\angle AOD = 6x$

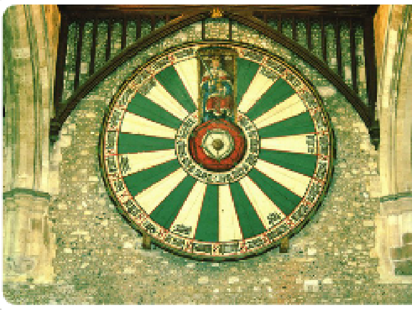
31. If $m\angle MQV = 90$, which expression can you use to find $m\angle VQP$?

Ⓐ $m\angle MQP - 90$ Ⓒ $m\angle MQP + 90$

Ⓑ $90 - m\angle MQV$ Ⓓ $90 + m\angle VQP$



32. Literature According to legend, King Arthur and his knights sat around the Round Table to discuss matters of the kingdom. The photo shows a round table on display at Winchester Castle, in England. From the center of the table, each section has the same degree measure. If King Arthur occupied two of these sections, what is the total degree measure of his section?



C Challenge

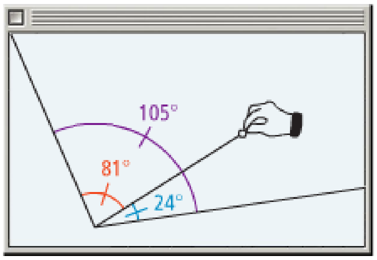
Time **Find the angle measure of the hands of a clock at each time.**

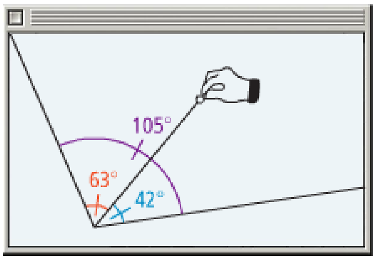
33. 6:00 **34.** 7:00 **35.** 11:00

36. 4:40 **37.** 5:20 **38.** 2:15

39. Open-Ended Sketch a right angle with vertex *V*. Name it ∠1. Then sketch a 135° angle that shares a side with ∠1. Name it ∠*PVB*. Is there more than one way to sketch ∠*PVB*? If so, sketch all the different possibilities. (*Hint:* Two angles are the same if you can rotate or flip one to match the other.)

40. **Technology** Your classmate constructs an angle. Then he constructs a ray from the vertex of the angle to a point in the interior of the angle. He measures all the angles formed. Then he moves the interior ray as shown below. What postulate do the two pictures support?

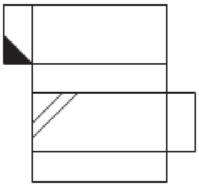


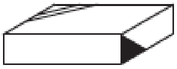


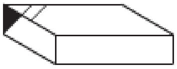
Standardized Test Prep

SAT/ACT

41. Which diagram shows the figure you can fold from the net at the right?



Ⓐ


Ⓑ


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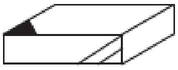

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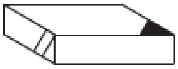


42. $\overline{XY}$ has endpoints $X = -72$ and $Y = 43$. What is XY?

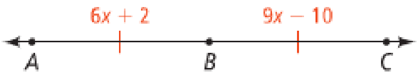
Ⓕ −115 Ⓖ −29 Ⓗ 29 Ⓘ 115

Short Response

43. Use the figure at the right.

a. What is the value of x?

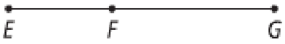
b. What is AC?



Mixed Review

Use the figure at the right.

See Lesson 1-3.



44. If $EG = 75$ and $EF = 28$, what is FG?

45. If $EG = 49$, $EF = 2x + 3$, and $FG = 4x - 2$, find x. Then find EF and FG.

Get Ready! **To prepare for Lesson 1-5, do Exercises 46–49.**

Algebra **Write and solve an equation to find the number(s).**

See p. 834.

46. Twice a number added to 4 is 28.

47. A number subtracted from 90 is three times that number.

48. The sum of two numbers is 180. One number is 5 times the other.

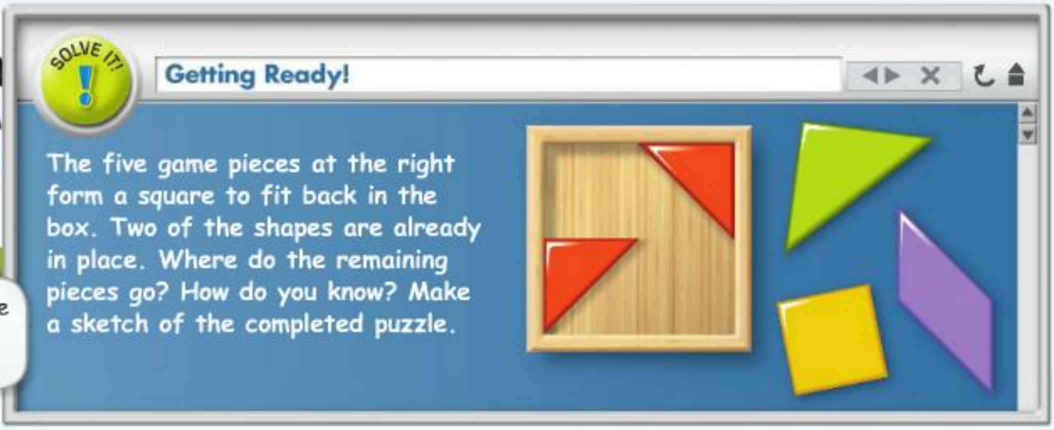
49. If $m\angle WXZ = 180$ and $m\angle ZXY = 115$, what is the measure of $\angle WXY$?

See Lesson 1-4.

1-5 Exploring Angle Pairs

Objective To identify special angle pairs and use their relationships to find angle measures

Look at the puzzle from different "angles"!


SOLVE IT!
Getting Ready!

The five game pieces at the right form a square to fit back in the box. Two of the shapes are already in place. Where do the remaining pieces go? How do you know? Make a sketch of the completed puzzle.

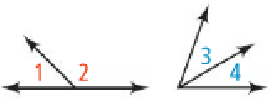
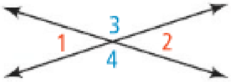
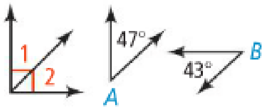
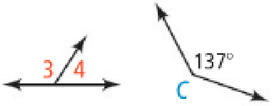
Lesson Vocabulary
- adjacent angles
- vertical angles
- complementary angles
- supplementary angles
- linear pair
- angle bisector

In this lesson, you will learn how to describe different kinds of angle pairs.

Essential Understanding Special angle pairs can help you identify geometric relationships. You can use these angle pairs to find angle measures.

take note

Key Concept Types of Angle Pairs

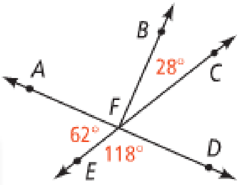
Definition	Example
Adjacent angles are two coplanar angles with a common side, a common vertex, and no common interior points.	$\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$ 
Vertical angles are two angles whose sides are opposite rays.	$\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$ 
Complementary angles are two angles whose measures have a sum of 90. Each angle is called the *complement* of the other.	$\angle 1$ and $\angle 2$, $\angle A$ and $\angle B$ 
Supplementary angles are two angles whose measures have a sum of 180. Each angle is called the *supplement* of the other.	$\angle 3$ and $\angle 4$, $\angle B$ and $\angle C$ 

Problem 1 Identifying Angle Pairs

Plan

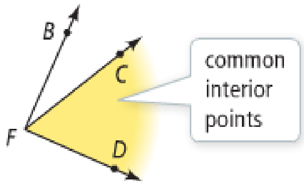
What should you look for in the diagram?
For part (A), check whether the angle pair matches every part of the definition of adjacent angles.

Use the diagram at the right. Is the statement true? Explain.



A **$\angle BFD$ and $\angle CFD$ are adjacent angles.**

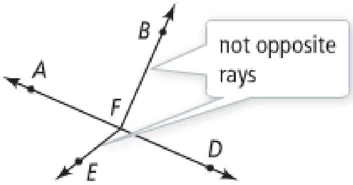
No. They have a common side ($\overrightarrow{FD}$) and a common vertex (F), but they also have common interior points. So $\angle BFD$ and $\angle CFD$ are not adjacent.



common interior points

B **$\angle AFB$ and $\angle EFD$ are vertical angles.**

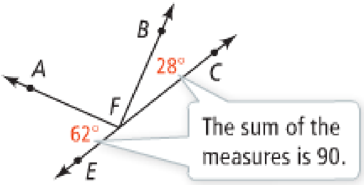
No. $\overrightarrow{FA}$ and $\overrightarrow{FD}$ are opposite rays, but $\overrightarrow{FE}$ and $\overrightarrow{FB}$ are not. So $\angle AFB$ and $\angle EFD$ are not vertical angles.



not opposite rays

C **$\angle AFE$ and $\angle BFC$ are complementary.**

Yes. $m\angle AFE + m\angle BFC = 62 + 28 = 90$. The sum of the angle measures is 90, so $\angle AFE$ and $\angle BFC$ are complementary.



The sum of the measures is 90.

Got It? **1.** Use the diagram in Problem 1. Is the statement true? Explain.

a. $\angle AFE$ and $\angle CFD$ are vertical angles.
b. $\angle BFC$ and $\angle DFE$ are supplementary.
c. $\angle BFD$ and $\angle AFB$ are adjacent angles.

take note

Concept Summary Finding Information From a Diagram

There are some relationships you can assume to be true from a diagram that has no marks or measures. There are other relationships you cannot assume directly. For example, you *can* conclude the following from an unmarked diagram.

- Angles are adjacent.
- Angles are adjacent and supplementary.
- Angles are vertical angles.

You *cannot* conclude the following from an unmarked diagram.

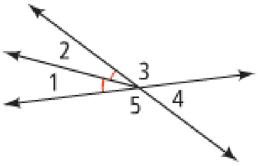
- Angles or segments are congruent.
- An angle is a right angle.
- Angles are complementary.

Think

How can you get information from a diagram?
Look for relationships between angles. For example, look for congruent angles and adjacent angles.

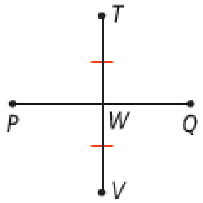
Problem 2 Making Conclusions From a Diagram

What can you conclude from the information in the diagram?



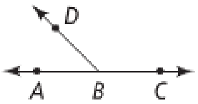
- $\angle 1 \cong \angle 2$ by the markings.
- $\angle 3$ and $\angle 5$ are vertical angles.
- $\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, $\angle 4$ and $\angle 5$, and $\angle 5$ and $\angle 1$ are adjacent angles.
- $\angle 3$ and $\angle 4$, and $\angle 4$ and $\angle 5$ are adjacent supplementary angles. So, $m\angle 3 + m\angle 4 = 180$ and $m\angle 4 + m\angle 5 = 180$ by the definition of supplementary angles.

Got It? **2.** Can you make each conclusion from the information in the diagram? Explain.



a. $\overline{TW} \cong \overline{WV}$
b. $\overline{PW} \cong \overline{WQ}$
c. $\angle TWQ$ is a right angle.
d. $\overline{TV}$ bisects $\overline{PQ}$.

A **linear pair** is a pair of adjacent angles whose noncommon sides are opposite rays. The angles of a linear pair form a straight angle.



take note

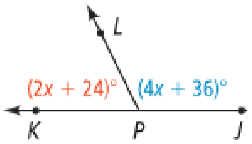
Postulate 1-9 Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.

Problem 3 Finding Missing Angle Measures

Algebra **$\angle KPL$ and $\angle JPL$ are a linear pair, $m\angle KPL = 2x + 24$, and $m\angle JPL = 4x + 36$. What are the measures of $\angle KPL$ and $\angle JPL$?**

Know	Need	Plan
$\angle KPL$ and $\angle JPL$ are supplementary.	$m\angle KPL$ and $m\angle JPL$	*Draw a diagram.* Use the definition of supplementary angles to write and solve an equation.



Step 1 $m\angle KPL + m\angle JPL = 180$ Def. of supplementary angles
$(2x + 24) + (4x + 36) = 180$ Substitute.
$6x + 60 = 180$ Combine like terms.
$6x = 120$ Subtract 60 from each side.
$x = 20$ Divide each side by 6.

Step 2 Evaluate the original expressions for $x = 20$.

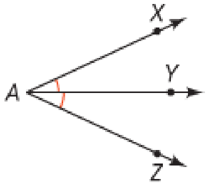
$m\angle KPL = 2x + 24 = 2 \cdot 20 + 24 = 40 + 24 = 64$
$m\angle JPL = 4x + 36 = 4 \cdot 20 + 36 = 80 + 36 = 116$

Substitute 20 for x.

Got It? **3. a.** **Reasoning** How can you check your results in Problem 3?

b. $\angle ADB$ and $\angle BDC$ are a linear pair. $m\angle ADB = 3x + 14$ and $m\angle BDC = 5x - 2$. What are $m\angle ADB$ and $m\angle BDC$?

An **angle bisector** is a ray that divides an angle into two congruent angles. Its endpoint is at the angle vertex. Within the ray, a segment with the same endpoint is also an angle bisector. The ray or segment bisects the angle. In the diagram, $\overrightarrow{AY}$ is the angle bisector of $\angle XAZ$, so $\angle XAY \cong \angle YAZ$.



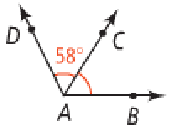
Problem 4 Using an Angle Bisector to Find Angle Measures

Multiple Choice $\overrightarrow{AC}$ **bisects** $\angle DAB$**. If** $m\angle DAC = 58$**, what is** $m\angle DAB$**?**

(A) 29 (B) 58 (C) 87 (D) 116

Plan

Draw a diagram to help you visualize what you are given and what you need to find.



Draw a diagram.

$m\angle CAB = m\angle DAC$	Definition of angle bisector
$= 58$	Substitute.
$m\angle DAB = m\angle CAB + m\angle DAC$	Angle Addition Postulate
$= 58 + 58$	Substitute.
$= 116$	Simplify.

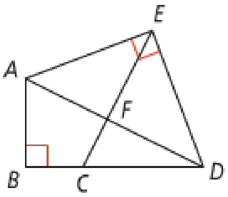
The measure of $\angle DAB$ is 116. The correct choice is D.

Got It? **4.** $\overrightarrow{KM}$ bisects $\angle JKL$. If $m\angle JKL = 72$, what is $m\angle JKM$?

Lesson Check

Do you know HOW?

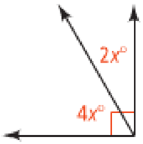
Name a pair of the following types of angle pairs.



1. vertical angles
2. complementary angles
3. linear pair
4. $\overrightarrow{PB}$ bisects $\angle RPT$ so that $m\angle RPB = x + 2$ and $m\angle TPB = 2x - 6$. What is $m\angle RPT$?

Do you UNDERSTAND?

5. **Vocabulary** How does the term *linear pair* describe how the angle pair looks?
6. **Error Analysis** Your friend calculated the value of x below. What is her error?



~~$4x + 2x = 180$~~
~~$6x = 180$~~
~~$x = 30$~~

Practice and Problem-Solving Exercises

A Practice

Use the diagram at the right. Is each statement true? Explain.

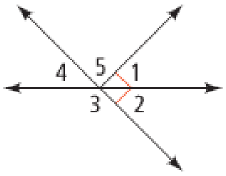
See Problem 1.

7. $\angle 1$ and $\angle 5$ are adjacent angles.

8. $\angle 3$ and $\angle 5$ are vertical angles.

9. $\angle 3$ and $\angle 4$ are complementary.

10. $\angle 1$ and $\angle 2$ are supplementary.



Name an angle or angles in the diagram described by each of the following.

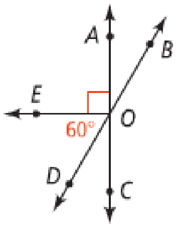
11. supplementary to $\angle AOD$

12. adjacent and congruent to $\angle AOE$

13. supplementary to $\angle EOA$

14. complementary to $\angle EOD$

15. a pair of vertical angles



For Exercises 16–23, can you make each conclusion from the information in the diagram? Explain.

See Problem 2.

16. $\angle J \cong \angle D$

17. $\angle JAC \cong \angle DAC$

18. $m\angle JCA = m\angle DCA$

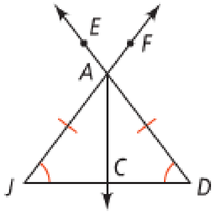
19. $m\angle JCA + m\angle ACD = 180$

20. $\overline{AJ} \cong \overline{AD}$

21. C is the midpoint of $\overline{JD}$.

22. $\angle JAE$ and $\angle EAF$ are adjacent and supplementary.

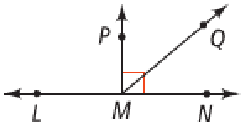
23. $\angle EAF$ and $\angle JAD$ are vertical angles.



See Problem 3.

24. Name two pairs of angles that form a linear pair in the diagram at the right.

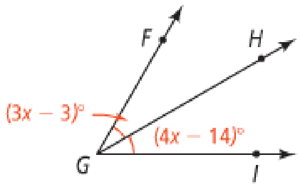
25. $\angle EFG$ and $\angle GFH$ are a linear pair, $m\angle EFG = 2n + 21$, and $m\angle GFH = 4n + 15$. What are $m\angle EFG$ and $m\angle GFH$?



See Problem 4.

26. **Algebra** In the diagram, $\overrightarrow{GH}$ bisects $\angle FGI$.

- **a.** Solve for x and find $m\angle FGH$.
- **b.** Find $m\angle HGI$.
- **c.** Find $m\angle FGI$.



B Apply

Algebra $\overrightarrow{BD}$ **bisects** $\angle ABC$**. Solve for** x **and find** $m\angle ABC$**.**

27. $m\angle ABD = 5x,\ m\angle DBC = 3x + 10$

28. $m\angle ABC = 4x - 12,\ m\angle ABD = 24$

29. $m\angle ABD = 4x - 16,\ m\angle CBD = 2x + 6$

30. $m\angle ABD = 3x + 20,\ m\angle CBD = 6x - 16$

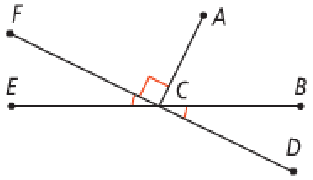
Algebra **Find the measure of each angle in the angle pair described.**

31. **Think About a Plan** The measure of one angle is twice the measure of its supplement.

- How many angles are there? What is their relationship?
- How can you use algebra, such as using the variable x, to help you?

32. The measure of one angle is 20 less than the measure of its complement.

In the diagram at the right, $m\angle ACB = 65$**. Find each of the following.**



33. $m\angle ACD$

34. $m\angle BCD$

35. $m\angle ECD$

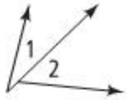
36. $m\angle ACE$

37. **Algebra** $\angle RQS$ and $\angle TQS$ are a linear pair where $m\angle RQS = 2x + 4$ and $m\angle TQS = 6x + 20$.

a. Solve for x.

b. Find $m\angle RQS$ and $m\angle TQS$.

c. Show how you can check your answer.



38. **Writing** In the diagram at the right, are $\angle 1$ and $\angle 2$ adjacent? Justify your reasoning.

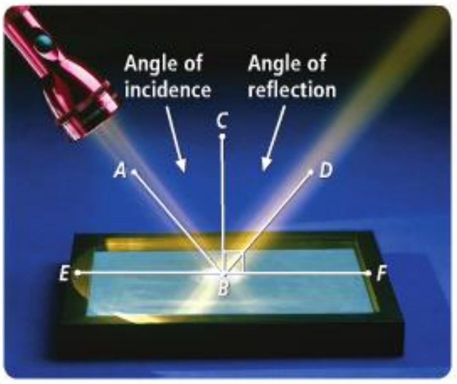
39. **Reasoning** When $\overrightarrow{BX}$ bisects $\angle ABC$, $\angle ABX \cong \angle CBX$. One student claims there is always a related equation $m\angle ABX = \frac{1}{2}m\angle ABC$. Another student claims the related equation is $2m\angle ABX = m\angle ABC$. Who is correct? Explain.

40. **Optics** A beam of light and a mirror can be used to study the behavior of light. Light that strikes the mirror is reflected so that the angle of reflection and the angle of incidence are congruent. In the diagram, $\angle ABC$ has a measure of 41.

a. Name the angle of reflection and find its measure.

b. Find $m\angle ABD$.

c. Find $m\angle ABE$ and $m\angle DBF$.



41. **Reasoning** Describe all situations where vertical angles are also supplementary.

Challenge

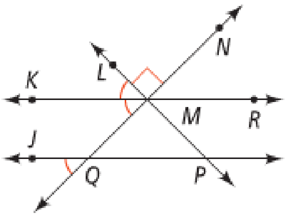
Name all of the angle(s) in the diagram described by the following.

42. supplementary to $\angle JQM$

43. adjacent and congruent to $\angle KMQ$

44. a linear pair with $\angle LMQ$

45. complementary to $\angle NMR$



46. **Coordinate Geometry** The x- and y-axes of the coordinate plane form four right angles. The interior of each of the right angles is a quadrant of the coordinate plane. What is the equation for the line that contains the angle bisector of Quadrants I and III?

47. $\overrightarrow{XC}$ bisects $\angle AXB$, $\overrightarrow{XD}$ bisects $\angle AXC$, $\overrightarrow{XE}$ bisects $\angle AXD$, $\overrightarrow{XF}$ bisects $\angle EXD$, $\overrightarrow{XG}$ bisects $\angle EXF$, and $\overrightarrow{XH}$ bisects $\angle DXB$. If $m\angle DXC = 16$, find $m\angle GXH$.

Standardized Test Prep

SAT/ACT

48. Which statement is true?

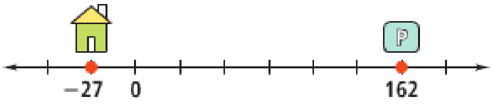
Ⓐ A right angle has a complement.

Ⓑ An obtuse angle has a complement.

Ⓒ The supplement of a right angle is a right angle.

Ⓓ Every angle has a supplement.

49. The diagram shows distance in meters. How far, in meters, is it from the parking lot to your house?



Ⓕ 44

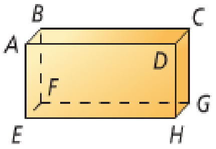
Ⓗ 183

Ⓖ 135

Ⓘ 189

Short Response

50. Draw a net for the box at the right. Label each corner with its corresponding letter. Some letters will be repeated. How can the repeated letters on the net help you visualize how the net folds into the solid?



Mixed Review

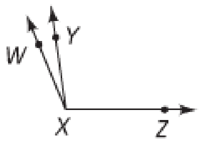
Use the diagram at the right.

See Lesson 1-4.

51. What is the acute angle?

52. What are the obtuse angles?

53. If $m\angle WXZ = 150$, $m\angle WXY = 8x - 1$, and $m\angle ZXY = 17x + 26$, what is $m\angle WXY$?



Get Ready! **To prepare for Lesson 1-6, do Exercises 54–59.**

Sketch each figure.

54. $\overrightarrow{GH}$

55. $\overline{CD}$

56. $\overleftrightarrow{AB}$

See Lesson 1-2.

57. acute $\angle ABC$

58. right $\angle PST$

59. straight $\angle XYZ$

See Lesson 1-4.

1 Mid-Chapter Quiz

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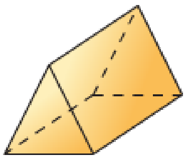
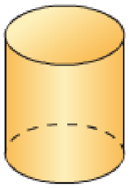
Do you know HOW?

Draw a net for each figure.

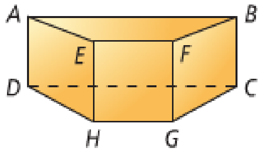
1.



2.



Determine whether the given points are coplanar. If *yes*, name the plane. If *no*, explain.



3. *A*, *E*, *F*, and *B*

4. *D*, *C*, *E*, and *F*

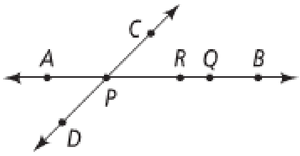
5. *H*, *G*, *F*, and *B*

6. *A*, *E*, *B*, and *C*

7. Use the figure from Exercises 3–6. Name the intersection of each pair of planes.

- **a.** plane *AEFB* and plane *CBFG*
- **b.** plane *EFGH* and plane *AEHD*

Use the figure below for Exercises 8–15.



8. Give two other names for $\overleftrightarrow{AB}$.

9. Give two other names for $\overrightarrow{PR}$.

10. Give two other names for $\angle CPR$.

11. Name three collinear points.

12. Name two opposite rays.

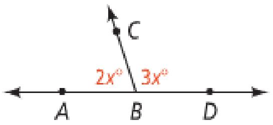
13. Name three segments.

14. Name two angles that form a linear pair.

15. Name a pair of vertical angles.

16. a. Algebra Find the value of *x* in the diagram below.

b. Classify $\angle ABC$ and $\angle CBD$ as *acute*, *right*, or *obtuse*.



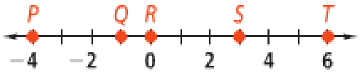
Find the length of each segment.

17. $\overline{PQ}$

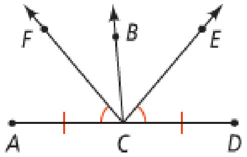
18. $\overline{RS}$

19. $\overline{ST}$

20. $\overline{QT}$



Use the figure below for Exercises 21–23.



21. Algebra If $AC = 4x + 5$ and $DC = 3x + 8$, find *AD*.

22. If $m\angle FCD = 130$ and $m\angle BCD = 95$, find $m\angle FCB$.

23. If $m\angle FCA = 50$, find $m\angle FCE$.

Do you UNDERSTAND?

24. Error Analysis Suppose $PQ = QR$. Your friend says that *Q* is always the midpoint of $\overline{PR}$. Is he correct? Explain.

25. Reasoning Determine whether the following situation is possible. Explain your reasoning. Include a sketch.

> Collinear points *C*, *F*, and *G* lie in plane *M*. $\overleftrightarrow{AB}$ intersects plane *M* at *C*. $\overleftrightarrow{AB}$ and $\overleftrightarrow{GF}$ do not intersect.

Concept Byte

Use With Lesson 1-6

ACTIVITY

Compass Designs

In Lesson 1-6, you will use a compass to construct geometric figures. You can construct figures to show geometric relationships, to suggest new relationships, or simply to make interesting geometric designs.

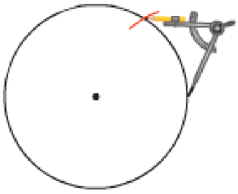
Activity

Step 1 Open your compass to about 2 in. Make a circle and mark the point at the center of the circle. Keep the opening of your compass fixed. Place the compass point on the circle. With the pencil end, make a small arc to intersect the circle.

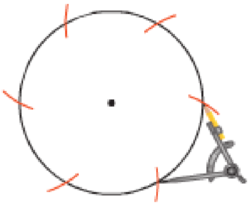
Step 2 Place the compass point on the circle at the arc. Mark another arc. Continue around the circle this way to draw four more arcs—six in all.

Step 3 Place your compass point on an arc you marked on the circle. Place the pencil end at the next arc. Draw a large arc that passes through the circle's center and continues to another point on the circle.

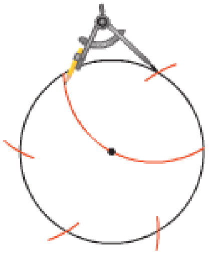
Step 4 Draw six large arcs in this manner, each centered at one of the six points marked on the circle. You may choose to color your design.



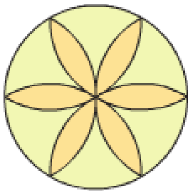
Step 1



Step 2



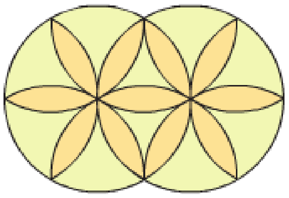
Step 3



Step 4

Exercises

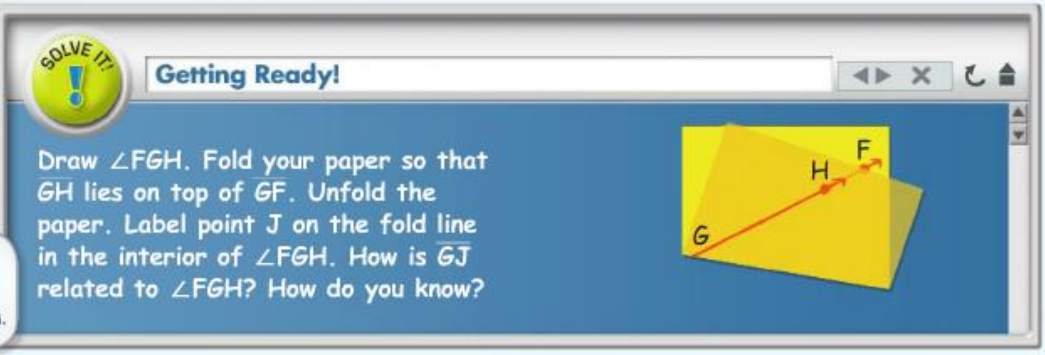
1. In Step 2, did your sixth mark on the circle land precisely on the point where you first placed your compass on the circle?
   a. Survey the class to find out how many did.
   b. Explain why your sixth mark may not have landed on your starting point.
2. Extend your design by using one of the six points on the circle as the center for a new circle. Repeat Steps 1–4 with this circle. Repeat several times to make interlocking circles



1-6 Basic Constructions

Objective To make basic constructions using a straightedge and a compass

You can compare angles even if you can't measure them.



Draw ∠FGH. Fold your paper so that $\overline{GH}$ lies on top of $\overline{GF}$. Unfold the paper. Label point J on the fold line in the interior of ∠FGH. How is $\overline{GJ}$ related to ∠FGH? How do you know?

In this lesson, you will learn another way to construct figures like the one above.

Lesson Vocabulary
- straightedge
- compass
- construction
- perpendicular lines
- perpendicular bisector

Essential Understanding You can use special geometric tools to make a figure that is congruent to an original figure without measuring. This method is more accurate than sketching and drawing.

A **straightedge** is a ruler with no markings on it. A **compass** is a geometric tool used to draw circles and parts of circles called *arcs*. A **construction** is a geometric figure drawn using a straightedge and a compass.

Problem 1 Constructing Congruent Segments

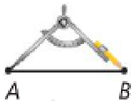
Construct a segment congruent to a given segment.

Given: $\overline{AB}$

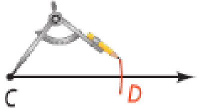
Construct: $\overline{CD}$ **so that** $\overline{CD} \cong \overline{AB}$

Step 1 Draw a ray with endpoint C.

Step 2 Open the compass to the length of $\overline{AB}$.



Step 3 With the same compass setting, put the compass point on point C. Draw an arc that intersects the ray. Label the point of intersection D.



$\overline{CD} \cong \overline{AB}$

Think

Why must the compass setting stay the same?
Using the same compass setting keeps segments congruent. It guarantees that the lengths of $\overline{AB}$ and $\overline{CD}$ are exactly the same.

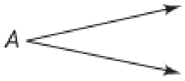
Got It? **1.** Use a straightedge to draw $\overline{XY}$. Then construct $\overline{RS}$ so that $RS = 2XY$.

Problem 2 Constructing Congruent Angles

Construct an angle congruent to a given angle.

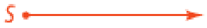
Given: $\angle A$

Construct: $\angle S$ **so that** $\angle S \cong \angle A$



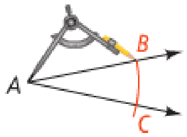
Step 1

Draw a ray with endpoint S.



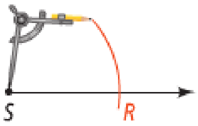
Step 2

With the compass point on vertex A, draw an arc that intersects the sides of $\angle A$. Label the points of intersection B and C.



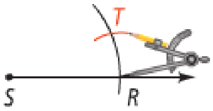
Step 3

With the same compass setting, put the compass point on point S. Draw an arc and label its point of intersection with the ray as R.



Step 4

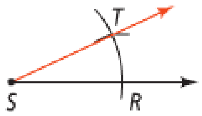
Open the compass to the length BC. Keeping the same compass setting, put the compass point on R. Draw an arc to locate point T.



Step 5

Draw $\overrightarrow{ST}$.

$\angle S \cong \angle A$



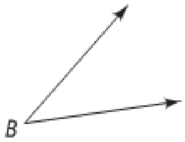
Think

Why do you need points like *B* and *C*?

B and *C* are reference points on the original angle. You can construct a congruent angle by locating corresponding points *R* and *T* on your new angle.

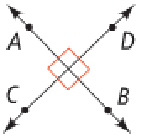
Got It? **2. a.** Construct $\angle F$ so that $m\angle F = 2m\angle B$.

b. **Reasoning** How is constructing a congruent angle similar to constructing a congruent segment?

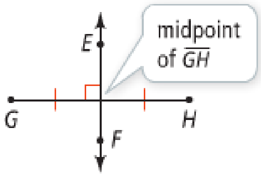


Dynamic Activity
Constructing Congruent Segments and Angles

Perpendicular lines are two lines that intersect to form right angles. The symbol $\perp$ means "is perpendicular to." In the diagram at the right, $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$.



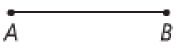
A **perpendicular bisector** of a segment is a line, segment, or ray that is perpendicular to the segment at its midpoint. In the diagram at the right, $\overleftrightarrow{EF}$ is the perpendicular bisector of $\overline{GH}$. The perpendicular bisector bisects the segment into two congruent segments. The construction in Problem 3 will show you how this works. You will justify the steps for this construction in Chapter 4, as well as for the other constructions in this lesson.



Problem 3 Constructing the Perpendicular Bisector

Construct the perpendicular bisector of a segment.

Given: $\overline{AB}$



Construct: $\overleftrightarrow{XY}$ so that $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$

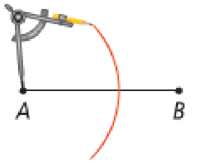
Think

Why must the compass opening be greater than $\frac{1}{2}AB$?

If the opening is less than $\frac{1}{2}AB$, the two arcs will not intersect in Step 2.

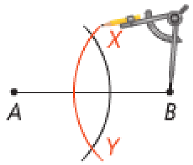
Step 1

Put the compass point on point A and draw a long arc as shown. Be sure the opening is greater than $\frac{1}{2}AB$.



Step 2

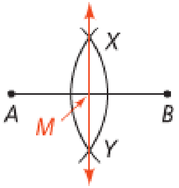
With the same compass setting, put the compass point on point B and draw another long arc. Label the points where the two arcs intersect as X and Y.



Step 3

Draw $\overleftrightarrow{XY}$. Label the point of intersection of $\overline{AB}$ and $\overleftrightarrow{XY}$ as M, the midpoint of $\overline{AB}$.

$\overleftrightarrow{XY} \perp \overline{AB}$ at midpoint M, so $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$.

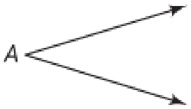


Got It? **3.** Draw $\overline{ST}$. Construct its perpendicular bisector.

Problem 4 Constructing the Angle Bisector

Construct the bisector of an angle.

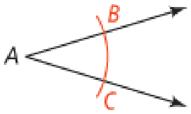
Given: $\angle A$



Construct: $\overrightarrow{AD}$, the bisector of $\angle A$

Step 1

Put the compass point on vertex A. Draw an arc that intersects the sides of $\angle A$. Label the points of intersection B and C.



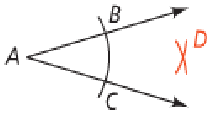
Think

Why must the arcs intersect?

The arcs need to intersect so that you have a point through which to draw a ray.

Step 2

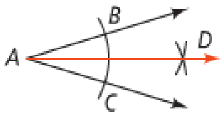
Put the compass point on point C and draw an arc. With the same compass setting, draw an arc using point B. Be sure the arcs intersect. Label the point where the two arcs intersect as D.



Step 3

Draw $\overrightarrow{AD}$.

$\overrightarrow{AD}$ is the bisector of $\angle CAB$.

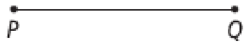


Got It? **4.** Draw obtuse $\angle XYZ$. Then construct its bisector $\overrightarrow{YP}$.

Lesson Check

Do you know **HOW?**

For Exercises 1 and 2, draw $\overline{PQ}$. Use your drawing as the original figure for each construction.



1. Construct a segment congruent to $\overline{PQ}$.

2. Construct the perpendicular bisector of $\overline{PQ}$.

3. Draw an obtuse $\angle JKL$. Construct its bisector.

Do you **UNDERSTAND?**

4. **Vocabulary** What two tools do you use to make constructions?

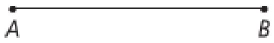
5. **Compare and Contrast** Describe the difference in accuracy between sketching a figure, drawing a figure with a ruler and protractor, and constructing a figure. Explain.

6. **Error Analysis** Your friend constructs $\overleftrightarrow{XY}$ so that it is perpendicular to and contains the midpoint of $\overline{AB}$. He claims that $\overline{AB}$ is the perpendicular bisector of $\overleftrightarrow{XY}$. What is his error?

Practice and Problem-Solving Exercises

A Practice

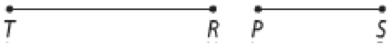
For Exercises 7–14, draw a diagram similar to the given one. Then do the construction. Check your work with a ruler or a protractor.



See Problem 1.

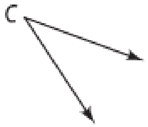
7. Construct $\overline{XY}$ congruent to $\overline{AB}$.

8. Construct $\overline{VW}$ so that $VW = 2AB$.



9. Construct $\overline{DE}$ so that $DE = TR + PS$.

10. Construct $\overline{QJ}$ so that $QJ = TR - PS$.



See Problem 2.

11. Construct $\angle D$ so that $\angle D \cong \angle C$.

12. Construct $\angle F$ so that $m\angle F = 2m\angle C$.

See Problem 3.

13. Construct the perpendicular bisector of $\overline{AB}$.

14. Construct the perpendicular bisector of $\overline{TR}$.

See Problem 4.

15. Draw acute $\angle PQR$. Then construct its bisector.

16. Draw obtuse $\angle XQZ$. Then construct its bisector.

B Apply

Sketch the figure described. Explain how to construct it. Then do the construction.

17. $\overleftrightarrow{XY} \perp \overleftrightarrow{YZ}$

18. $\overrightarrow{ST}$ bisects right $\angle PSQ$.

19. **Compare and Contrast** How is constructing an angle bisector similar to constructing a perpendicular bisector?

20. **Think About a Plan** Draw an $\angle A$. Construct an angle whose measure is $\frac{1}{4}m\angle A$.

- How is the angle you need to construct related to the angle bisector of $\angle A$?
- How can you use previous constructions to help you?

21. Answer the questions about a segment in a plane. Explain each answer.

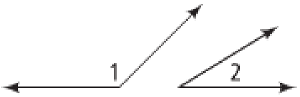
a. How many midpoints does the segment have?
b. How many bisectors does it have?
c. How many lines in the plane are its perpendicular bisectors?
d. How many lines in space are its perpendicular bisectors?

For Exercises 22–24, copy ∠1 and ∠2. Construct each angle described.

22. $\angle B$; $m\angle B = m\angle 1 + m\angle 2$

23. $\angle C$; $m\angle C = m\angle 1 - m\angle 2$

24. $\angle D$; $m\angle D = 2m\angle 2$



25. **Writing** Explain how to do each construction with a compass and straightedge.

a. Draw a segment $\overline{PQ}$. Construct the midpoint of $\overline{PQ}$.
b. Divide $\overline{PQ}$ into four congruent segments.

26. a. Draw a large triangle with three acute angles. Construct the bisectors of the three angles. What appears to be true about the three angle bisectors?

b. Repeat the constructions with a triangle that has one obtuse angle.

c. **Make a Conjecture** What appears to be true about the three angle bisectors of any triangle?

Use a ruler to draw segments of 2 cm, 4 cm, and 5 cm. Then construct each triangle with the given side measures, if possible. If it is not possible, explain why not.

27. 4 cm, 4 cm, and 5 cm

28. 2 cm, 5 cm, and 5 cm

29. 2 cm, 2 cm, and 5 cm

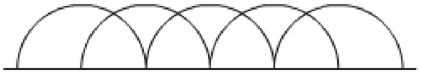
30. 2 cm, 2 cm, and 4 cm

31. a. Draw a segment, $\overline{XY}$. Construct a triangle with sides congruent to $\overline{XY}$.

b. Measure the angles of the triangle.

c. **Writing** Describe how to construct a 60° angle using what you know. Then describe how to construct a 30° angle.

32. Which steps best describe how to construct the pattern at the right?



Ⓐ Use a straightedge to draw the segment and then a compass to draw five half circles.

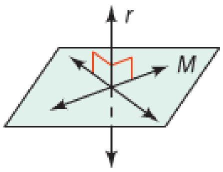
Ⓑ Use a straightedge to draw the segment and then a compass to draw six half circles.

Ⓒ Use a compass to draw five half circles and then a straightedge to join their ends.

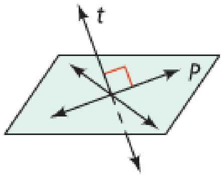
Ⓓ Use a compass to draw six half circles and then a straightedge to join their ends.

Challenge

33. Study the figures. Complete the definition of a line perpendicular to a plane: A line is perpendicular to a plane if it is __?__ to every line in the plane that __?__.



Line $r \perp$ plane M.



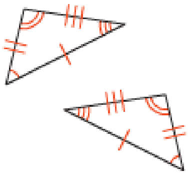
Line t is not $\perp$ plane P.

34. a. Use your compass to draw a circle. Locate three points A, B, and C on the circle.

b. Draw $\overline{AB}$ and $\overline{BC}$. Then construct the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$.

c. **Reasoning** Label the intersection of the two perpendicular bisectors as point O. What do you think is true about point O?

35. Two triangles are *congruent* if each side and each angle of one triangle is congruent to a side or angle of the other triangle. In Chapter 4, you will learn that if each side of one triangle is congruent to a side of the other triangle, then you can conclude that the triangles are congruent without finding the angles. Explain how you can use congruent triangles to justify the angle bisector construction.



Standardized Test Prep

SAT/ACT

36. What must you do to construct the midpoint of a segment?

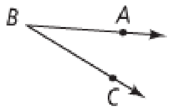
Ⓐ Measure half its length.

Ⓑ Construct an angle bisector.

Ⓒ Measure twice its length.

Ⓓ Construct a perpendicular bisector.

37. Given the diagram at the right, what is NOT a reasonable name for the angle?



Ⓕ $\angle ABC$

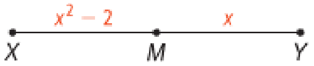
Ⓖ $\angle B$

Ⓗ $\angle CBA$

Ⓘ $\angle ACB$

Short Response

38. M is the midpoint of $\overline{XY}$. Find the value of x. Show your work.



Mixed Review

39. $\angle DEF$ is the supplement of $\angle DEG$ with $m\angle DEG = 64$. What is $m\angle DEF$? **See Lesson 1-5.**

40. $m\angle TUV = 100$ and $m\angle VUW = 80$. Are $\angle TUV$ and $\angle VUW$ a linear pair? Explain.

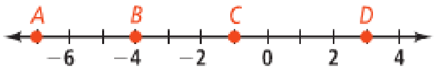
Find the length of each segment. **See Lesson 1-3.**

41. $\overline{AC}$

42. $\overline{AD}$

43. $\overline{CD}$

44. $\overline{BC}$



Get Ready! **To prepare for Lesson 1-7, do Exercises 45–47.**

Algebra Evaluate each expression for $a = 6$ and $b = -8$. **See p. 830.**

45. $(a - b)^2$

46. $\sqrt{a^2 + b^2}$

47. $\frac{a + b}{2}$

Concept Byte

Use With Lesson 1-6

TECHNOLOGY

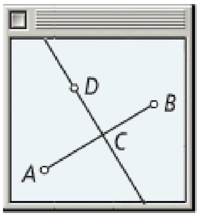
Exploring Constructions

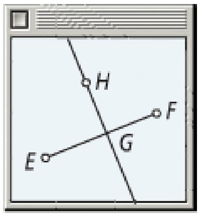
You can use Draw tools or Construct tools in geometry software to make points, lines, and planes. A figure made by Draw has no constraints. When you manipulate, or try to change, a figure made by Draw, it moves or changes size freely. A figure made by Construct is related to an existing object. When you manipulate the existing object, the constructed object moves or resizes accordingly.

In this Activity, you will explore the difference between Draw and Construct.

Activity

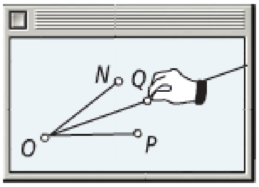
Draw $\overline{AB}$ and Construct the perpendicular bisector $\overleftrightarrow{DC}$. Then Draw $\overline{EF}$ and Construct G, any point on $\overline{EF}$. Draw $\overleftrightarrow{HG}$.



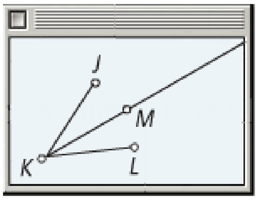


1. Find EG, GF, and $m\angle HGF$. Try to drag G so that $EG = GF$. Try to drag H so that $m\angle HGF = 90$. Were you able to draw the perpendicular bisector of $\overline{EF}$? Explain.
2. Drag A and B. Observe AC, CB, and $m\angle DCB$. Is $\overleftrightarrow{DC}$ always the perpendicular bisector of $\overline{AB}$ no matter how you manipulate the figure?
3. Drag E and F. Observe EG, GF, and $m\angle HGF$. How is the relationship between $\overline{EF}$ and $\overleftrightarrow{HG}$ different from the relationship between $\overline{AB}$ and $\overleftrightarrow{DC}$?
4. Write a description of the general difference between Draw and Construct. Then use your description to explain why the relationship between $\overline{EF}$ and $\overleftrightarrow{HG}$ differs from the relationship between $\overline{AB}$ and $\overleftrightarrow{DC}$.

Exercises



5. a. Draw $\angle NOP$. Draw $\overrightarrow{OQ}$ in the interior of $\angle NOP$. Drag Q until $m\angle NOQ = m\angle QOP$.
   b. Manipulate the figure and observe the different angle measures. Is $\overrightarrow{OQ}$ always the angle bisector of $\angle NOP$?



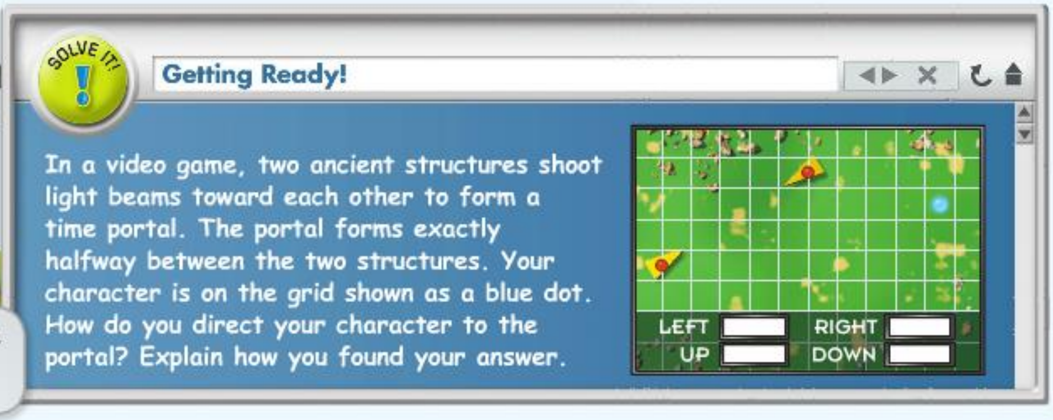
6. a. Draw $\angle JKL$.
   b. Construct its angle bisector, $\overrightarrow{KM}$.
   c. Manipulate the figure and observe the different angle measures. Is $\overrightarrow{KM}$ always the angle bisector of $\angle JKL$?
   d. How can you manipulate the figure on the screen so that it shows a right angle? Justify your answer.

1-7

Midpoint and Distance in the Coordinate Plane

Objectives To find the midpoint of a segment
To find the distance between two points in the coordinate plane

You can only move in horizontal and vertical units.



Getting Ready!

In a video game, two ancient structures shoot light beams toward each other to form a time portal. The portal forms exactly halfway between the two structures. Your character is on the grid shown as a blue dot. How do you direct your character to the portal? Explain how you found your answer.

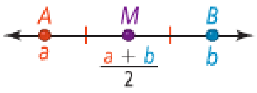
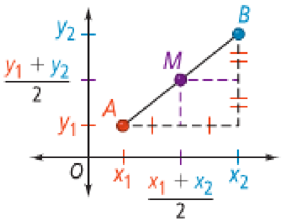
In this lesson, you will learn how to find midpoints and distance on a grid like the one in the Solve It.

Dynamic Activity
Finding Distance

Essential Understanding You can use formulas to find the midpoint and length of any segment in the coordinate plane.

take note

Key Concept Midpoint Formulas

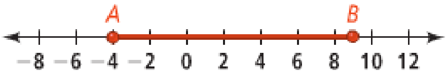
Description	Formula	Diagram
On a Number Line The coordinate of the midpoint is the *average* or *mean* of the coordinates of the endpoints.	The coordinate of the midpoint M of $\overline{AB}$ is $\frac{a+b}{2}$.	
In the Coordinate Plane The coordinates of the midpoint are the average of the x-coordinates and the average of the y-coordinates of the endpoints.	Given $\overline{AB}$ where $A(x_1, y_1)$ and $B(x_2, y_2)$, the coordinates of the midpoint of $\overline{AB}$ are $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.	 B M A y_2 $\frac{y_1+y_2}{2}$ y_1 O x_1 $\frac{x_1+x_2}{2}$ x_2

Problem 1 Finding the Midpoint

Think

Which Midpoint Formula do you use?
If the endpoints are real numbers, use the formula for the number line. If they are ordered pairs, use the formula for the coordinate plane.

A **$\overline{AB}$ has endpoints at -4 and 9. What is the coordinate of its midpoint?**

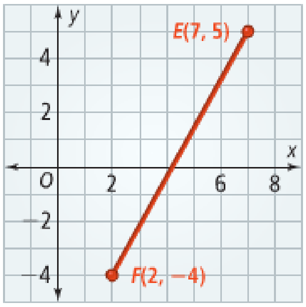


Let $a = -4$ and $b = 9$.

$$M = \frac{a+b}{2} = \frac{-4+9}{2} = \frac{5}{2} = 2.5$$

The coordinate of the midpoint of $\overline{AB}$ is 2.5.

B **$\overline{EF}$ has endpoints $E(7, 5)$ and $F(2, -4)$. What are the coordinates of its midpoint M?**



Let $E(7, 5)$ be (x_1, y_1) and $F(2, -4)$ be (x_2, y_2).

$$x\text{-coordinate of } M = \frac{x_1 + x_2}{2} = \frac{7+2}{2} = \frac{9}{2} = 4.5$$

$$y\text{-coordinate of } M = \frac{y_1 + y_2}{2} = \frac{5+(-4)}{2} = \frac{1}{2} = 0.5$$

The coordinates of the midpoint of $\overline{EF}$ are $M(4.5, 0.5)$.

Got It? **1. a.** $\overline{JK}$ has endpoints at -12 and 4 on a number line. What is the coordinate of its midpoint?

b. What is the midpoint of $\overline{RS}$ with endpoints $R(5, -10)$ and $S(3, 6)$?

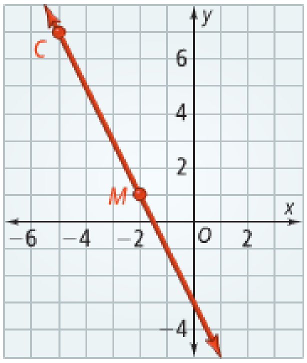
When you know the midpoint and an endpoint of a segment, you can use the Midpoint Formula to find the other endpoint.

Problem 2 Finding an Endpoint

Plan

How can you find the coordinates of *D*?
Use the Midpoint Formula to set up an equation. Split that equation into two equations: one for the *x*-coordinate and one for the *y*-coordinate.

The midpoint of $\overline{CD}$ is $M(-2, 1)$. One endpoint is $C(-5, 7)$. What are the coordinates of the other endpoint D?



Let $M(-2, 1)$ be (x, y) and $C(-5, 7)$ be (x_1, y_1). Let the coordinates of D be (x_2, y_2).

$$(-2, 1) = \left(\frac{-5 + x_2}{2}, \frac{7 + y_2}{2}\right)$$

x		y
$-2 = \frac{-5 + x_2}{2}$	Use the Midpoint Formula.	$1 = \frac{7 + y_2}{2}$
$-4 = -5 + x_2$	Multiply each side by 2.	$2 = 7 + y_2$
$1 = x_2$	Simplify.	$-5 = y_2$

The coordinates of D are $(1, -5)$.

Got It? **2.** The midpoint of $\overline{AB}$ has coordinates $(4, -9)$. Endpoint A has coordinates $(-3, -5)$. What are the coordinates of B?

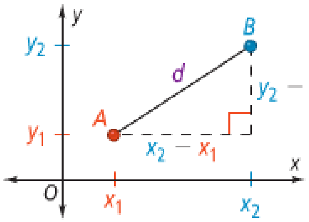
In Lesson 1-3, you learned how to find the distance between two points on a number line. To find the distance between two points in a coordinate plane, you can use the Distance Formula.

take note

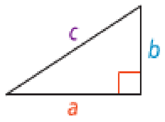
Key Concept Distance Formula

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



The Distance Formula is based on the *Pythagorean Theorem*, which you will study later in this book. When you use the Distance Formula, you are really finding the length of a side of a right triangle. You will verify the Distance Formula in Chapter 8.



$a^2 + b^2 = c^2$

Problem 3 Finding Distance

GRIDDED RESPONSE

What is the distance between $U(-7, 5)$ and $V(4, -3)$? Round to the nearest tenth.

Let $U(-7, 5)$ be (x_1, y_1) and $V(4, -3)$ be (x_2, y_2).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Use the Distance Formula.

$= \sqrt{(4 - (-7))^2 + (-3 - 5)^2}$ Substitute.

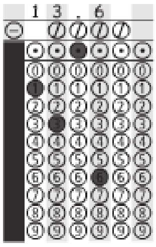
$= \sqrt{(11)^2 + (-8)^2}$ Simplify within the parentheses.

$= \sqrt{121 + 64}$ Simplify.

$= \sqrt{185}$

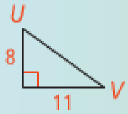
185 √ 13.60147051 Use a calculator.

To the nearest tenth, $UV = 13.6$.



Think

What part of a right triangle is $\overline{UV}$?
$\overline{UV}$ is the hypotenuse of a right triangle with legs of length 11 and 8.

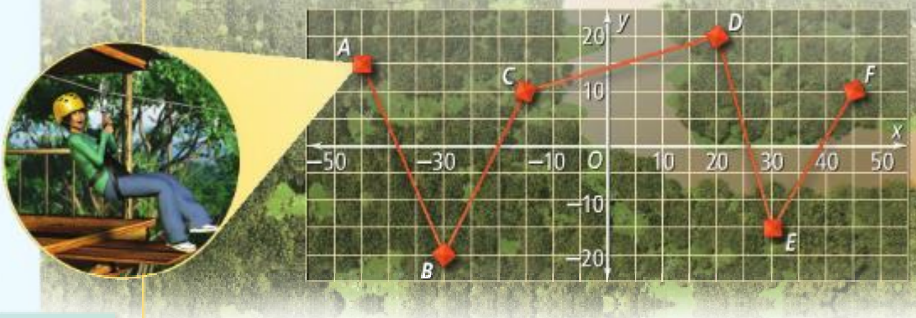


Got It? **3. a.** $\overline{SR}$ has endpoints $S(-2, 14)$ and $R(3, -1)$. What is SR to the nearest tenth?

b. Reasoning In Problem 3, suppose you let $V(4, -3)$ be (x_1, y_1) and $U(-7, 5)$ be (x_2, y_2). Do you get the same result? Why?

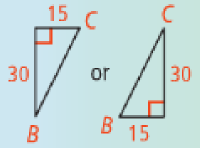
Problem 4 Finding Distance

Recreation **On a zip-line course, you are harnessed to a cable that travels through the treetops. You start at Platform *A* and zip to each of the other platforms. How far do you travel from Platform *B* to Platform *C*? Each grid unit represents 5 m.**



Think

Where's the right triangle?
The lengths of the legs of the right triangle are 15 and 30. There are two possibilities:



Let Platform $B(-30, -20)$ be (x_1, y_1) and Platform $C(-15, 10)$ be (x_2, y_2).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ Use the Distance Formula.

$$= \sqrt{(-15 - (-30))^2 + (10 - (-20))^2}$$ Substitute.

$$= \sqrt{15^2 + 30^2} = \sqrt{225 + 900} = \sqrt{1125}$$ Simplify.

√ 1125 = 33.54101966 Use a calculator.

You travel about 33.5 m from Platform *B* to Platform *C*.

Got It? **4.** How far do you travel from Platform *D* to Platform *E*?

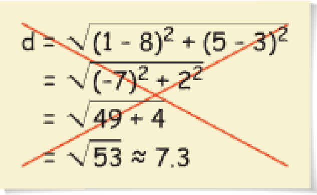
Lesson Check

Do you know HOW?

1. $\overline{RS}$ has endpoints $R(2, 4)$ and $S(-1, 7)$. What are the coordinates of its midpoint *M*?
2. The midpoint of $\overline{BC}$ is $(5, -2)$. One endpoint is $B(3, 4)$. What are the coordinates of endpoint *C*?
3. What is the distance between points $K(-9, 8)$ and $L(-6, 0)$?

Do you UNDERSTAND?

4. **Reasoning** How does the Distance Formula ensure that the distance between two different points is positive?
5. **Error Analysis** Your friend calculates the distance between points $Q(1, 5)$ and $R(3, 8)$. What is his error?



$d = \sqrt{(1 - 8)^2 + (5 - 3)^2}$
$= \sqrt{(-7)^2 + 2^2}$
$= \sqrt{49 + 4}$
$= \sqrt{53} \approx 7.3$

Practice and Problem-Solving Exercises

A Practice

Find the coordinate of the midpoint of the segment with the given endpoints. **See Problem 1.**

6. 2 and 4

7. −9 and 6

8. 2 and −5

9. −8 and −12

Find the coordinates of the midpoint of $\overline{HX}$.

10. $H(0, 0), X(8, 4)$

11. $H(-1, 3), X(7, -1)$

12. $H(13, 8), X(-6, -6)$

13. $H(7, 10), X(5, -8)$

14. $H(-6.3, 5.2), X(1.8, -1)$

15. $H\left(5\frac{1}{2}, -4\frac{3}{4}\right), X\left(2\frac{1}{4}, -1\frac{1}{4}\right)$

The coordinates of point T are given. The midpoint of $\overline{ST}$ is $(5, -8)$. Find the coordinates of point S. **See Problem 2.**

16. $T(0, 4)$

17. $T(5, -15)$

18. $T(10, 18)$

19. $T(-2, 8)$

20. $T(1, 12)$

21. $T(4.5, -2.5)$

Find the distance between each pair of points. If necessary, round to the nearest tenth. **See Problem 3.**

22. $J(2, -1), K(2, 5)$

23. $L(10, 14), M(-8, 14)$

24. $N(-1, -11), P(-1, -3)$

25. $A(0, 3), B(0, 12)$

26. $C(12, 6), D(-8, 18)$

27. $E(6, -2), F(-2, 4)$

28. $Q(12, -12), T(5, 12)$

29. $R(0, 5), S(12, 3)$

30. $X(-3, -4), Y(5, 5)$

Maps **For Exercises 31–35, use the map below. Find the distance between the cities to the nearest tenth.** **See Problem 4.**

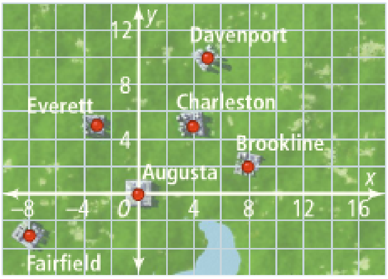
31. Augusta and Brookline

32. Brookline and Charleston

33. Brookline and Davenport

34. Everett and Fairfield

35. List the cities in the order of least to greatest distance from Augusta.



B Apply

Find (a) PQ to the nearest tenth and (b) the coordinates of the midpoint of $\overline{PQ}$.

36. $P(3, 2), Q(6, 6)$

37. $P(0, -2), Q(3, 3)$

38. $P(-4, -2), Q(1, 3)$

39. $P(-5, 2), Q(0, 4)$

40. $P(-3, -1), Q(5, -7)$

41. $P(-5, -3), Q(-3, -5)$

42. $P(-4, -5), Q(-1, 1)$

43. $P(2, 3), Q(4, -2)$

44. $P(4, 2), Q(3, 0)$

45. **Think About a Plan** An airplane at $T(80, 20)$ needs to fly to both $U(20, 60)$ and $V(110, 85)$. What is the shortest possible distance for the trip? Explain.

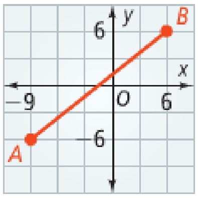
- What type of information do you need to find the shortest distance?
- How can you use a diagram to help you?

46. **Reasoning** The midpoint of $\overline{TS}$ is the origin. Point T is located in Quadrant II. What quadrant contains point S? Explain.

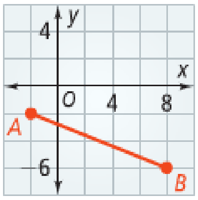
47. Do you use the Midpoint Formula or the Distance Formula to find the following?
  a. Given points K and P, find the distance from K to the midpoint of $\overline{KP}$.
  b. Given point K and the midpoint of $\overline{KP}$, find KP.

For each graph, find (a) AB to the nearest tenth and (b) the coordinates of the midpoint of $\overline{AB}$.

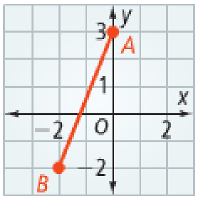
48.



49.

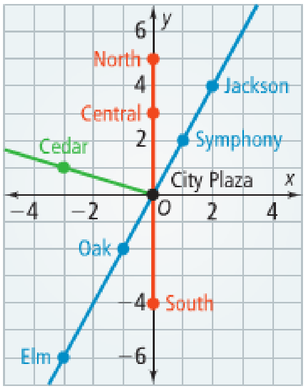


50.



51. **Coordinate Geometry** Graph the points $A(2, 1)$, $B(6, -1)$, $C(8, 7)$, and $D(4, 9)$. Draw parallelogram $ABCD$, and diagonals $\overline{AC}$ and $\overline{BD}$.
  a. Find the midpoints of $\overline{AC}$ and $\overline{BD}$.
  b. What appears to be true about the diagonals of a parallelogram?

Travel The units of the subway map at the right are in miles. Suppose the routes between stations are straight. Find the distance you would travel between each pair of stations to the nearest tenth of a mile.



52. Oak Station and Jackson Station

53. Central Station and South Station

54. Elm Station and Symphony Station

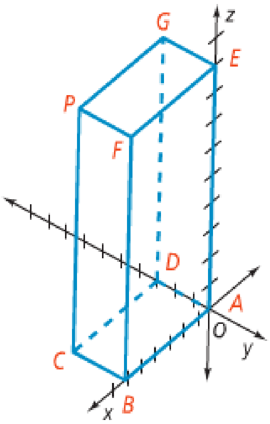
55. Cedar Station and City Plaza Station

56. Maple Station is located 6 mi west and 2 mi north of City Plaza. What is the distance between Cedar Station and Maple Station?

57. **Open-Ended** Point $H(2, 2)$ is the midpoint of many segments.
  a. Find the coordinates of the endpoints of four noncollinear segments that have point H as their midpoint.
  b. You know that a segment with midpoint H has length 8. How many possible noncollinear segments match this description? Explain.

Challenge

58. Points $P(-4, 6)$, $Q(2, 4)$, and R are collinear. One of the points is the midpoint of the segment formed by the other two points.
  a. What are the possible coordinates of R?
  b. **Reasoning** $RQ = \sqrt{160}$. Does this information affect your answer to part (a)? Explain.



Geometry in 3 Dimensions **You can use three coordinates (x, y, z) to locate points in three dimensions.**

59. Point P has coordinates $(6, -3, 9)$ as shown at the right. Give the coordinates of points A, B, C, D, E, F, and G.

Distance in 3 Dimensions **In a three-dimensional coordinate system, you can find the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) with this extension of the Distance Formula.**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Find the distance between each pair of points to the nearest tenth.

60. $P(2, 3, 4)$, $Q(-2, 4, 9)$

61. $T(0, 12, 15)$, $V(-8, 20, 12)$

Standardized Test Prep

SAT/ACT

62. A segment has endpoints $(14, -8)$ and $(4, 12)$. What are the coordinates of its midpoint?

Ⓐ $(9, 10)$

Ⓑ $(-5, 10)$

Ⓒ $(5, -10)$

Ⓓ $(9, 2)$

63. Which of these is the first step in constructing a congruent segment?

Ⓕ Draw a ray.

Ⓖ Find the midpoint.

Ⓗ Label two points.

Ⓘ Measure the segment.

Short Response

64. The midpoint of $\overline{RS}$ is $N(-4, 1)$. One endpoint is $S(0, -7)$.

a. What are the coordinates of R?

b. What is the length of $\overline{RS}$ to the nearest tenth of a unit?

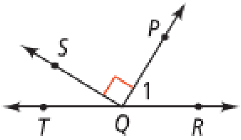
Mixed Review

Use a straightedge and a compass. **See Lesson 1-6.**

65. Draw $\overline{AB}$. Construct $\overline{PQ}$ so that $PQ = 2AB$.

66. Draw an acute $\angle RTS$. Construct the bisector of $\angle RTS$.

Use the diagram at the right. **See Lesson 1-4.**



67. Name $\angle 1$ two other ways.

68. If $m\angle PQR = 60$, what is $m\angle RQS$?

Get Ready! **To prepare for Lesson 1-8, do Exercises 69–72.**

Complete each statement. Use the conversion table on page 837. **See p. 826.**

69. 130 in. = ■ ft

70. 14 yd = ■ in.

71. 27 ft = ■ yd

72. 2 mi = ■ ft

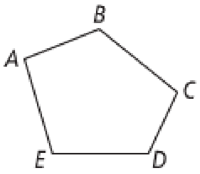
Review

Use With Lesson 1-8

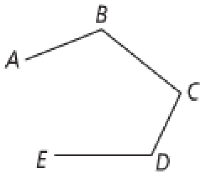
Classifying Polygons

In geometry, a figure that lies in a plane is called a *plane figure*.

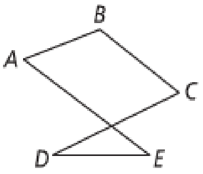
A **polygon** is a closed plane figure formed by three or more segments. Each segment intersects exactly two other segments at their endpoints. No two segments with a common endpoint are collinear. Each segment is called a *side*. Each endpoint of a side is a *vertex*.



A polygon



Not a polygon;
not a closed figure



Not a polygon;
two sides intersect
between endpoints.

To name a polygon, start at any vertex and list the vertices consecutively in a clockwise or counterclockwise direction.

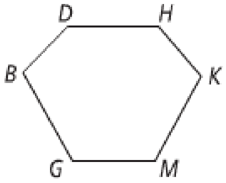
Example 1

Name the polygon. Then identify its sides and angles.

Two names for this polygon are *DHKMGB* and *MKHDBG*.

sides: $\overline{DH}$, $\overline{HK}$, $\overline{KM}$, $\overline{MG}$, $\overline{GB}$, $\overline{BD}$

angles: $\angle D$, $\angle H$, $\angle K$, $\angle M$, $\angle G$, $\angle B$

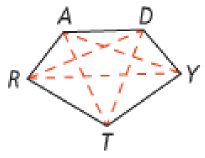


You can classify a polygon by its number of sides. The tables below show the names of some common polygons.

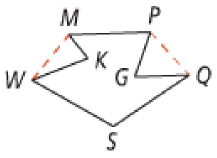
Names of Common Polygons

Sides	Name	Sides	Name
3	Triangle, or trigon	9	Nonagon, or enneagon
4	Quadrilateral, or tetragon	10	Decagon
5	Pentagon	11	Hendecagon
6	Hexagon	12	Dodecagon
7	Heptagon	⋮	⋮
8	Octagon	*n*	*n*-gon

You can also classify a polygon as concave or convex, using the diagonals of the polygon. A **diagonal** is a segment that connects two nonconsecutive vertices.



A **convex polygon** has no diagonal with points outside the polygon.

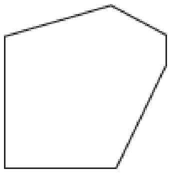


A **concave polygon** has at least one diagonal with points outside the polygon.

In this textbook, a polygon is convex unless otherwise stated.

Example 2

Classify the polygon by its number of sides. Tell whether the polygon is *convex* or *concave*.



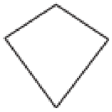
The polygon has six sides. Therefore, it is a hexagon.

No diagonal of the hexagon contains points outside the hexagon. The hexagon is convex.

Exercises

Is the figure a polygon? If not, explain why.

1.



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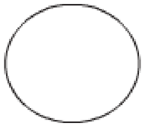

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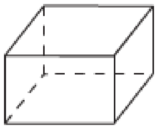

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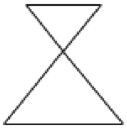
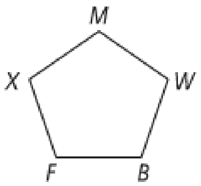


Name the polygon. Then identify its sides and angles.

5.



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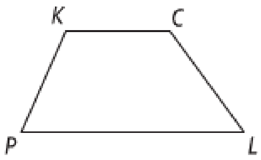

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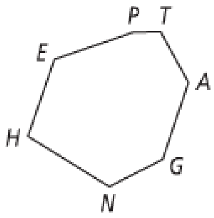
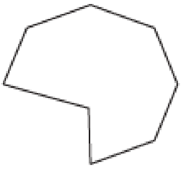


Classify the polygon by its number of sides. Tell whether the polygon is *convex* or *concave*.

8.



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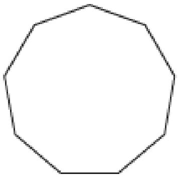

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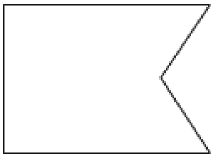
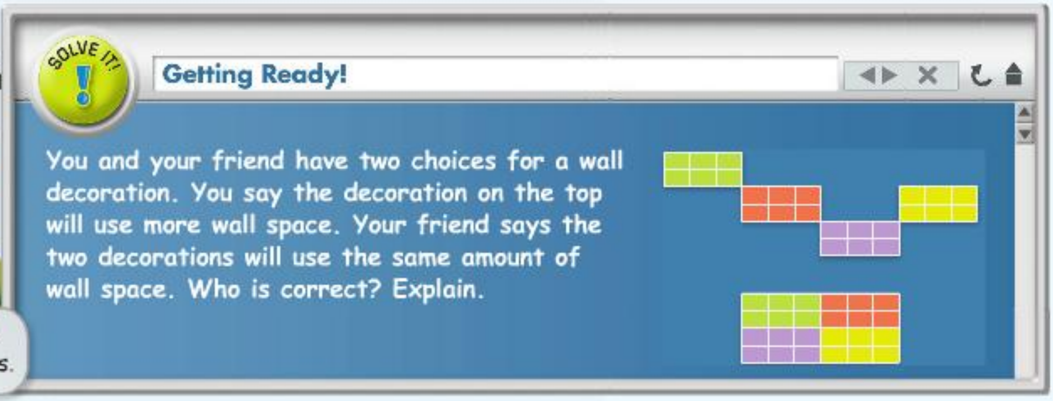


1-8

Perimeter, Circumference, and Area

Objectives To find the perimeter or circumference of basic shapes
To find the area of basic shapes

Think about what "wall space" means.



Getting Ready!

You and your friend have two choices for a wall decoration. You say the decoration on the top will use more wall space. Your friend says the two decorations will use the same amount of wall space. Who is correct? Explain.

Dynamic Activity
Perimeter, Circumference, and Area

Lesson Vocabulary
- perimeter
- area

In the Solve It, you considered various ideas of what it means to take up space on a flat surface.

Essential Understanding Perimeter and area are two different ways of measuring geometric figures.

The **perimeter** P of a polygon is the sum of the lengths of its sides. The **area** A of a polygon is the number of square units it encloses. For figures such as squares, rectangles, triangles, and circles, you can use formulas for perimeter (or *circumference* C for circles) and area.

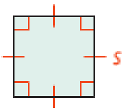
take note

Key Concept Perimeter, Circumference, and Area

Square
side length s

$P = 4s$

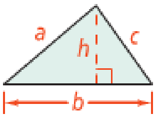
$A = s^2$



Triangle
side lengths a, b, and c, base b, and height h

$P = a + b + c$

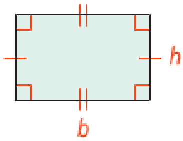
$A = \frac{1}{2}bh$



Rectangle
base b and height h

$P = 2b + 2h$, or $2(b + h)$

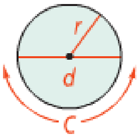
$A = bh$



Circle
radius r and diameter d

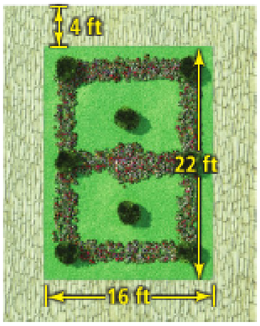
$C = \pi d$, or $C = 2\pi r$

$A = \pi r^2$



The units of measurement for perimeter and circumference include inches, feet, yards, miles, centimeters, and meters. When measuring area, use square units such as square inches (in.^{2}), square feet (ft^2), square yards (yd^2), square miles (mi^2), square centimeters (cm^2), and square meters (m^2).

Problem 1 Finding the Perimeter of a Rectangle

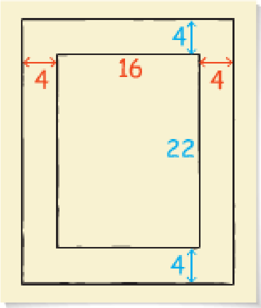


Landscaping **The botany club members are designing a rectangular garden for the courtyard of your school. They plan to place edging on the outside of the path. How much edging material will they need?**

Plan

Why should you *draw a diagram*?
A diagram can help you see the larger rectangle formed by the garden and the path, and which lengths to add together.

Step 1 Find the dimensions of the garden, including the path.



For a rectangle, "length" and "width" are sometimes used in place of "base" and "height."

Width of the garden and path

$= 4 + 16 + 4 = 24$

Length of the garden and path

$= 4 + 22 + 4 = 30$

Step 2 Find the perimeter of the garden including the path.

$P = 2b + 2h$ Use the formula for the perimeter of a rectangle.

$= 2(24) + 2(30)$ Substitute 24 for b and 30 for h.

$= 48 + 60$ Simplify.

$= 108$

You will need 108 ft of edging material.

Got It? **1.** You want to frame a picture that is 5 in. by 7 in. with a 1-in.-wide frame.

a. What is the perimeter of the picture?

b. What is the perimeter of the outside edge of the frame?

You can name a circle with the symbol ⊙. For example, the circle with center A is written ⊙A.

The formulas for a circle involve the special number *pi* (π). Pi is the ratio of any circle's circumference to its diameter. Since π is an irrational number,

$$\pi = 3.1415926\ldots,$$

you cannot write it as a terminating decimal. For an approximate answer, you can use 3.14 or $\frac{22}{7}$ for π. You can also use the π key on your calculator to get a rounded decimal for π. For an exact answer, leave the result in terms of π.

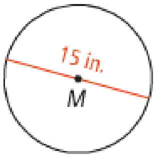
Problem 2 Finding Circumference

Plan

Which formula should you use?
You want to find circumference. Since you know the diameter in part (A), it would be easier to use the circumference formula that involves diameter.

What is the circumference of the circle in terms of π? What is the circumference of the circle to the nearest tenth?

A $\odot M$



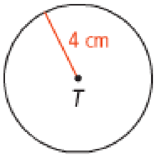
$C = \pi d$ Use the formula for circumference of a circle.

$= \pi(15)$ This is the exact answer.

≈ 47.1238898 Use a calculator.

The circumference of $\odot M$ is 15π in., or about 47.1 in.

B $\odot T$



$C = 2\pi r$ Use the formula for circumference of a circle.

$= 2\pi(4)$ This is the exact answer.

$= 8\pi$ Simplify.

≈ 25.13274123 Use a calculator.

The circumference of $\odot T$ is 8π cm, or about 25.1 cm.

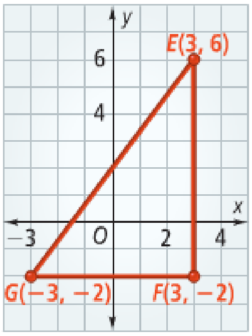
Got It? **2. a.** What is the circumference of a circle with radius 24 m in terms of π?
b. What is the circumference of a circle with diameter 24 m to the nearest tenth?

Problem 3 Finding Perimeter in the Coordinate Plane

Plan

What do you need?
To find the perimeter of a figure, you need its side lengths. Use what you know about length on a number line and in the coordinate plane.

Coordinate Geometry **What is the perimeter of $\triangle EFG$?**



Step 1 Find the length of each side.

$EF = |6 - (-2)| = 8$
$FG = |3 - (-3)| = 6$ Use the Ruler Postulate.

$EG = \sqrt{(3 - (-3))^2 + (6 - (-2))^2}$ Use the Distance Formula.

$= \sqrt{6^2 + 8^2}$ Simplify within the parentheses.

$= \sqrt{36 + 64}$ Simplify.

$= \sqrt{100}$

$= 10$

Step 2 Add the side lengths to find the perimeter.

$EF + FG + EG = 8 + 6 + 10 = 24$

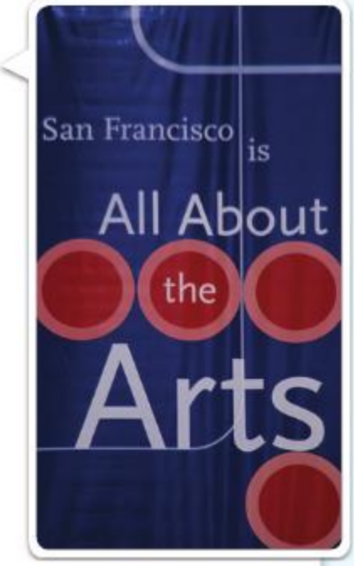
The perimeter of $\triangle EFG$ is 24 units.

Got It? **3.** Graph quadrilateral $JKLM$ with vertices $J(-3, -3)$, $K(1, -3)$, $L(1, 4)$, and $M(-3, 1)$. What is the perimeter of $JKLM$?

To find area, you should use the same unit for both dimensions.

Problem 4 Finding Area of a Rectangle

Banners **You want to make a rectangular banner similar to the one at the right. The banner shown is $2\frac{1}{2}$ ft wide and 5 ft high. To the nearest square yard, how much material do you need?**



Think

How can you check your conversion?
Yards are longer than feet, so the number you get in yards should be less than the given number in feet. Since $\frac{5}{6} < 2\frac{1}{2}$, the conversion checks.

Step 1 Convert the dimensions of the banner to yards. Use the conversion factor $\frac{1\text{ yd}}{3\text{ ft}}$.

$$\text{Width: } \frac{5}{2}\cancel{\text{ft}} \cdot \frac{1\text{ yd}}{3\cancel{\text{ft}}} = \frac{5}{6}\text{ yd} \qquad 2\frac{1}{2}\text{ ft} = \frac{5}{2}\text{ ft}$$

$$\text{Height: } 5\cancel{\text{ft}} \cdot \frac{1\text{ yd}}{3\cancel{\text{ft}}} = \frac{5}{3}\text{ yd}$$

Step 2 Find the area of the banner.

$A = bh$ Use the formula for area of a rectangle.

$= \frac{5}{6} \cdot \frac{5}{3}$ Substitute $\frac{5}{6}$ for b and $\frac{5}{3}$ for h.

$= \frac{25}{18}$

The area of the banner is $\frac{25}{18}$, or $1\frac{7}{18}$ square yards (yd^2). You need 2 yd^2 of material.

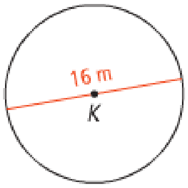
Got It? **4.** You are designing a poster that will be 3 yd wide and 8 ft high. How much paper do you need to make the poster? Give your answer in square feet.

Problem 5 Finding Area of a Circle

Plan

What are you given?
In circle problems, make it a habit to note whether you are given the radius or the diameter. In this case, you are given the diameter.

What is the area of $\odot K$ in terms of π?



Step 1 Find the radius of $\odot K$.

$r = \frac{16}{2}$, or 8 The radius is half the diameter.

Step 2 Use the radius to find the area.

$A = \pi r^2$ Use the formula for area of a circle.

$= \pi(8)^2$ Substitute 8 for r.

$= 64\pi$ Simplify.

The area of $\odot K$ is 64π m^2.

Got It? **5.** The diameter of a circle is 14 ft.

a. What is the area of the circle in terms of π?

b. What is the area of the circle using an approximation of π?

c. **Reasoning** Which approximation of π did you use in part (b)? Why?

The following postulate is useful in finding areas of figures with irregular shapes.

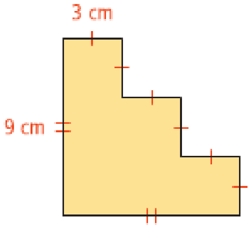
take note

Postulate 1-10 Area Addition Postulate

The area of a region is the sum of the areas of its nonoverlapping parts.

Problem 6 Finding Area of an Irregular Shape

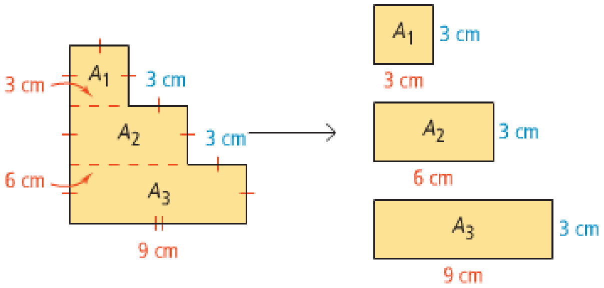
Multiple Choice What is the area of the figure at the right? All angles are right angles.



(A) 27 cm^2 (C) 45 cm^2

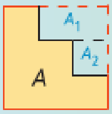
(B) 36 cm^2 (D) 54 cm^2

Step 1 Separate the figure into rectangles.



Think

What is another way to find the area? Extend the figure to form a square. Then subtract the areas of basic shapes from the area of the square.



$A = A_{\text{square}} - A_1 - A_2$

Step 2 Find A_1, A_2, and A_3.

$\text{Area} = bh$ Use the formula for the area of a rectangle.

$A_1 = 3 \cdot 3 = 9$ Substitute for the base and height.

$A_2 = 6 \cdot 3 = 18$

$A_3 = 9 \cdot 3 = 27$

Step 3 Find the total area of the figure.

$\text{Total Area} = A_1 + A_2 + A_3$ Use the Area Addition Postulate.

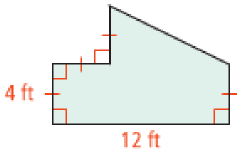
$= 9 + 18 + 27$

$= 54$

The area of the figure is 54 cm^2. The correct choice is D.

Got It? **6. a. Reasoning** What is another way to separate the figure in Problem 6?

b. What is the area of the figure at the right?



Lesson Check

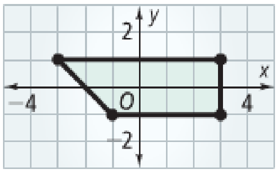
Do you know HOW?

1. What is the perimeter and area of a rectangle with base 3 in. and height 7 in.?

2. What is the circumference and area of each circle to the nearest tenth?

a. $r = 9$ in. **b.** $d = 7.3$ m

3. What is the perimeter and area of the figure at the right?



Do you UNDERSTAND?

4. **Writing** Describe a real-world situation in which you would need to find a perimeter. Then describe a situation in which you would need to find an area.

5. **Compare and Contrast** Your friend can't remember whether $2\pi r$ computes the circumference or the area of a circle. How would you help your friend? Explain.

6. **Error Analysis** A classmate finds the area of a circle with radius 30 in. to be 900 in.^{2}. What error did your classmate make?

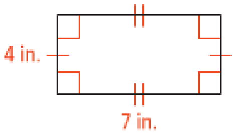
Practice and Problem-Solving Exercises

A Practice

Find the perimeter of each figure.

See Problem 1.

7.



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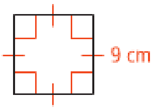
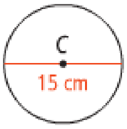


9. **Fencing** A garden that is 5 ft by 6 ft has a walkway 2 ft wide around it. What is the amount of fencing needed to surround the walkway?

Find the circumference of $\odot C$ in terms of π.

See Problem 2.

10.



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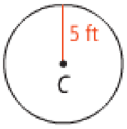

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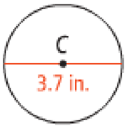

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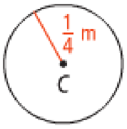


Coordinate Geometry **Graph each figure in the coordinate plane. Find each perimeter.**

See Problem 3.

14. $X(0, 2)$, $Y(4, -1)$, $Z(-2, -1)$

15. $A(-4, -1)$, $B(4, 5)$, $C(4, -2)$

16. $L(0, 1)$, $M(3, 5)$, $N(5, 5)$, $P(5, 1)$

17. $S(-5, 3)$, $T(7, -2)$, $U(7, -6)$, $V(-5, -6)$

Find the area of each rectangle with the given base and height.

See Problem 4.

18. 4 ft, 4 in.

19. 30 in., 4 yd

20. 2 ft 3 in., 6 in.

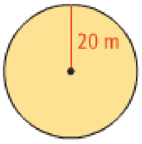
21. 40 cm, 2 m

22. **Roads** What is the area of a section of pavement that is 20 ft wide and 100 yd long? Give your answer in square feet.

Find the area of each circle in terms of π.

See Problem 5.

23.



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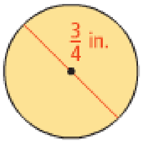

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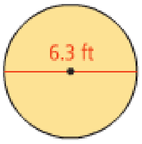

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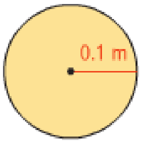


Find the area of each circle using an approximation of π. If necessary, round to the nearest tenth.

27. $r = 7$ ft

28. $d = 8.3$ m

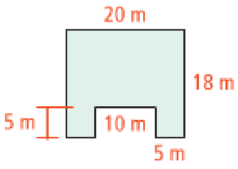
29. $d = 24$ cm

30. $r = 12$ in.

Find the area of the shaded region. All angles are right angles.

See Problem 6.

31.



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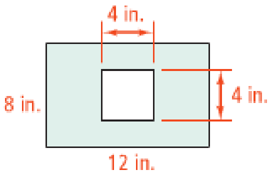

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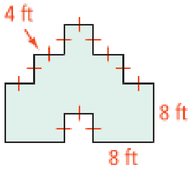


B Apply

Home Maintenance **To determine how much of each item to buy, tell whether you need to know area or perimeter. Explain your choice.**

34. wallpaper for a bedroom

35. crown molding for a ceiling

36. fencing for a backyard

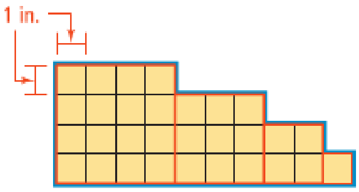
37. paint for a basement floor

38. **Think About a Plan** A light-year unit describes the distance that one photon of light travels in one year. The Milky Way galaxy has a diameter of about 100,000 light-years. The distance to Earth from the center of the Milky Way galaxy is about 30,000 light-years. How many more light-years does a star on the outermost edge of the Milky Way travel in one full revolution around the galaxy compared to Earth?

- What do you know about the shape of each orbital path?
- Are you looking for circumference or area?
- How do you compare the paths using algebraic expressions?

39. **a.** What is the area of a square with sides 12 in. long? 1 ft long?
b. How many square inches are in a square foot?

40. **a.** Count squares at the right to find the area of the polygon outlined in blue.
b. Use a formula to find the area of each square outlined in red.
c. **Writing** How does the sum of your results in part (b) compare to your result in part (a)? Which postulate does this support?



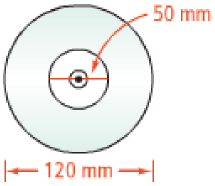
41. The area of an 11-cm-wide rectangle is 176 cm^2. What is its length?

42. A square and a rectangle have equal areas. The rectangle is 64 cm by 81 cm. What is the perimeter of the square?

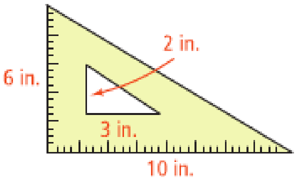
43. A rectangle has perimeter 40 cm and base 12 cm. What is its area?

Find the area of each shaded figure.

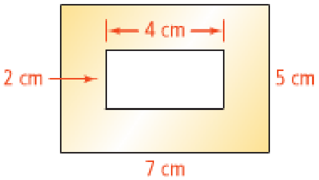
44. compact disc



45. drafting triangle



46. picture frame

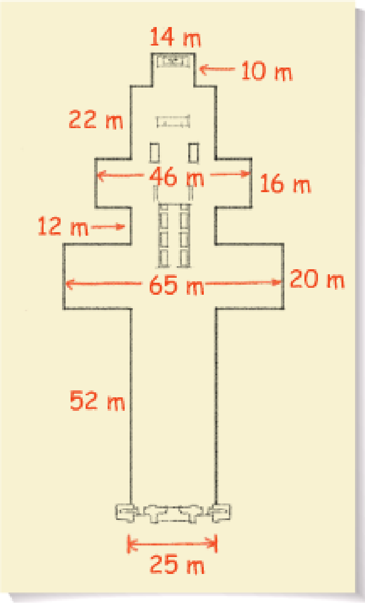


47. a. **Reasoning** Can you use the formula for the perimeter of a rectangle to find the perimeter of any square? Explain.

b. Can you use the formula for the perimeter of a square to find the perimeter of any rectangle? Explain.

c. Use the formula for the perimeter of a square to write a formula for the area of a square in terms of its perimeter.

48. **Estimation** On an art trip to England, a student sketches the floor plan of the main body of Salisbury Cathedral. The shape of the floor plan is called the building's "footprint." The student estimates the dimensions of the cathedral on her sketch at the right. Use the student's lengths to estimate the area of Salisbury Cathedral's footprint.



49. **Coordinate Geometry** The endpoints of a diameter of a circle are $A(2, 1)$ and $B(5, 5)$. Find the area of the circle in terms of π.

50. **Algebra** A rectangle has a base of x units. The area is $(4x^2 - 2x)$ square units. What is the height of the rectangle in terms of x?

Ⓐ $(4 - x)$ units

Ⓑ $(x - 2)$ units

Ⓒ $(4x^3 - 2x^2)$ units

Ⓓ $(4x - 2)$ units

Coordinate Geometry **Graph each rectangle in the coordinate plane. Find its perimeter and area.**

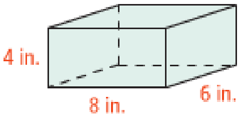
51. $A(-3, 2)$, $B(-2, 2)$, $C(-2, -2)$, $D(-3, -2)$

52. $A(-2, -6)$, $B(-2, -3)$, $C(3, -3)$, $D(3, -6)$

53. The surface area of a three-dimensional figure is the sum of the areas of all of its surfaces. You can find the surface area by finding the area of a net for the figure.

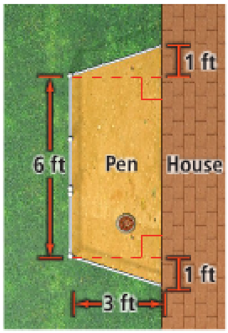
a. Draw a net for the solid shown. Label the dimensions.

b. What is the area of the net? What is the surface area of the solid?



54. **Coordinate Geometry** On graph paper, draw polygon $ABCDEFG$ with vertices $A(1, 1)$, $B(10, 1)$, $C(10, 8)$, $D(7, 5)$, $E(4, 5)$, $F(4, 8)$, and $G(1, 8)$. Find the perimeter and the area of the polygon.

55. **Pet Care** You want to adopt a puppy from your local animal shelter. First, you plan to build an outdoor playpen along the side of your house, as shown on the right. You want to lay down special dog grass for the pen's floor. If dog grass costs $1.70 per square foot, how much will you spend?



56. A rectangular garden has an 8-ft walkway around it. How many more feet is the outer perimeter of the walkway than the perimeter of the garden?

Challenge

Algebra **Find the area of each figure.**

57. a rectangle with side lengths $\frac{2a}{5b}$ units and $\frac{3b}{8}$ units

58. a square with perimeter $10n$ units

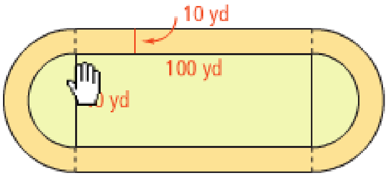
59. a triangle with base $(5x - 2y)$ units and height $(4x + 3y)$ units

Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

60. An athletic field is a 100 yd-by-40 yd rectangle with a semicircle at each of the short sides. A running track 10 yd wide surrounds the field. Find the perimeter of the outside of the running track to the nearest tenth of a yard.



61. A square garden has a 4-ft walkway around it. The garden has a perimeter of 260 ft. What is the area of the walkway in square feet?

62. $A(4, -1)$ and $B(-2, 3)$ are points in a coordinate plane. M is the midpoint of $\overline{AB}$. What is the length of $\overline{MB}$ to the nearest tenth of a unit?

63. Find CD to the nearest tenth if point C is at $(12, -8)$ and point D is at $(5, 19)$.

Mixed Review

Find (a) AB to the nearest tenth and (b) the midpoint coordinates of $\overline{AB}$. **See Lesson 1-7.**

64. $A(4, 1)$, $B(7, 9)$ **65.** $A(0, 3)$, $B(-3, 8)$ **66.** $A(-1, 1)$, $B(-4, -5)$

$\overleftrightarrow{BG}$ is the perpendicular bisector of $\overline{WR}$ at point K. **See Lesson 1-6.**

67. What is $m\angle BKR$?

68. Name two congruent segments.

Get Ready! **To prepare for Lesson 2-1, do Exercise 69.**

69. **a.** Copy and extend this list to show the first 10 perfect squares. **See p. 829.**

$$1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, \ldots$$

b. Which do you think describes the square of any odd number?

It is odd. It is even.

Concept Byte

Use With Lesson 1-8

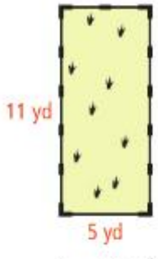
TECHNOLOGY

Comparing Perimeters and Areas

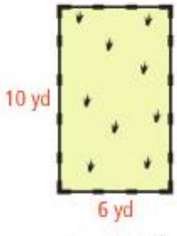
You can use a graphing calculator or spreadsheet software to find maximum and minimum values. These values help you solve real-world problems where you want to minimize or maximize a quantity such as cost or time. In this Activity, you will find minimum and maximum values for area and perimeter problems.

Activity

You have 32 yd of fencing. You want to make a rectangular horse pen with maximum area.



$A = 55\text{ yd}^2$

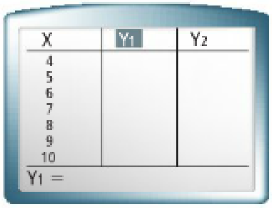


$A = 60\text{ yd}^2$

1. Draw some possible rectangular pens and find their areas. Use the examples at the right as models.

2. You plan to use all of your fencing. Let X represent the base of the pen. What is the height of the pen in terms of X? What is the area of the pen in terms of X?

3. Make a graphing calculator table to find area. Again, let X represent the base. For Y_1, enter the expression you wrote for the height in Question 2. For Y_2, enter the expression you wrote for the area in Question 2. Set the table so that X starts at 4 and changes by 1. Scroll down the table.
  a. What value of X gives you the maximum area?
  b. What is the maximum area?



X	Y1	Y2
4		
5		
6		
7		
8		
9		
10		

Y1 =

4. Use your calculator to graph Y_2. Describe the shape of the graph. Trace on the graph to find the coordinates of the highest point. What is the relationship, if any, between the coordinates of the highest point on the graph and your answers to Question 3? Explain.

Exercises

5. For a fixed perimeter, what rectangular shape will result in a maximum area?

6. Consider that the pen is not limited to polygon shapes. What is the area of a circular pen with circumference 32 yd? How does this result compare to the maximum area you found in the Activity?

7. You plan to make a rectangular garden with an area of 900 ft^2. You want to use a minimum amount of fencing to keep the cost low.
  a. List some possible dimensions for the garden. Find the perimeter of each.
  b. Make a graphing calculator table. Use integer values of the base b, and the corresponding values of the height h, to find values for P, the perimeter. What dimensions will give you a garden with the minimum perimeter?

1 Pull It All Together

To solve these problems, you will pull together many concepts and skills that you learned in this chapter. They are the basic tools used to study geometry.

BIG idea Visualization

You can draw and construct figures to understand geometric relationships.

Task 1

Graph $\triangle ABC$ with $A(4, 7)$, $B(0,0)$, and $C(8, 1)$.

a. Which sides of $\triangle ABC$ are congruent? How do you know?

b. Construct the bisector of $\angle B$. Mark the intersection of the ray and $\overline{AC}$ as D.

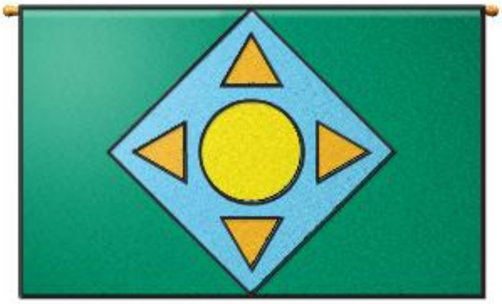
c. What do you notice about AD and CD?

BIG idea Measurement

Measures of segments and angles give you ways to describe geometric figures.

Task 2

Using the design below, you plan to make a rectangular wall hanging to decorate your bedroom. The green rectangle is 16 ft by 10 ft, the blue square is 7 ft by 7 ft, the orange triangles have a height of 1.73 ft and a side length of 2 ft, and the yellow circle has a diameter of 3 ft. You plan to glue the pieces together and then outline each piece with black cord.



a. How much black cord does the design use?

b. How much fabric of each color does the design show?

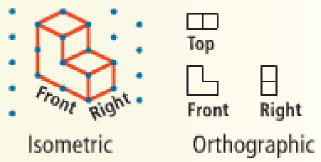
c. At a craft store, fabric comes in bolts that are a fixed number of inches wide. You can only buy fabric by the whole yard. If each bolt is 48 in. wide, what is the least amount of each color that you need to buy? Explain.

1 Chapter Review

Connecting BIG ideas and Answering the Essential Questions

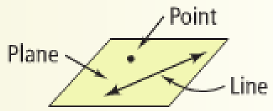
1 Visualization
You can represent a 3-D figure with a 2-D drawing by visualizing the surfaces of the figure and how they relate to each other.

Lesson 1-1 Nets and Drawings



2 Reasoning
Geometry is a mathematical system built on basic terms, definitions, and assumptions called postulates.

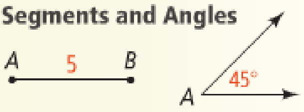
Lesson 1-2 Points, Lines, and Planes



Lesson 1-5 Exploring Angle Pairs
You can conclude these relationships from an unmarked diagram.
- Angles are adjacent.
- Angles are adjacent supplementary.
- Angles are vertical angles.

3 Measurement
You can describe the attributes of a segment or angle by using unit amounts.

Lessons 1-3 and 1-4 Segments and Angles



$AB = 5$ $m\angle A = 45$

Lesson 1-7 Midpoint and Distance

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Chapter Vocabulary

- acute, right, obtuse, straight angles (p. 29)
- adjacent angles (p. 34)
- angle bisector (p. 37)
- collinear points, coplanar (p. 12)
- complementary angles (p. 34)
- congruent angles (p. 29)
- congruent segments (p. 22)
- construction (p. 43)
- isometric drawing (p. 5)
- linear pair (p. 36)
- measure of an angle (p. 28)
- net (p. 4)
- orthographic drawing (p. 6)
- perpendicular bisector (p. 44)
- perpendicular lines (p. 44)
- point, line, plane (p. 11)
- postulate, axiom (p. 13)
- ray, opposite rays (p. 12)
- segment (p. 12)
- segment bisector (p. 22)
- space (p. 12)
- supplementary angles (p. 34)
- vertex of an angle (p. 27)
- vertical angles (p. 34)

Choose the correct term to complete each sentence.

1. A ray that divides an angle into two congruent angles is a(n) _?_.

2. _?_ are two lines that intersect to form right angles.

3. A(n) _?_ is a two-dimensional diagram that you can fold to form a 3-D figure.

4. _?_ are two angles with measures that have a sum of 90.

1-1 Nets and Drawings for Visualizing Geometry

Quick Review

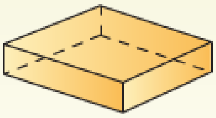
A **net** is a two-dimensional pattern that you can fold to form a three-dimensional figure. A net shows all surfaces of a figure in one view.

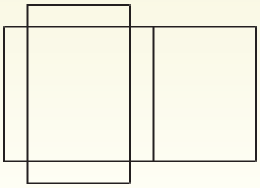
An **isometric drawing** shows a corner view of a three-dimensional object. It allows you to see the top, front, and side of the object in one view.

An **orthographic drawing** shows three separate views of a three-dimensional object: a top view, a front view, and a right-side view.

Example

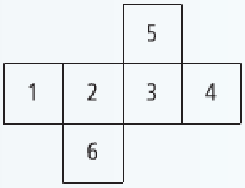
Draw a net for the solid at the right.



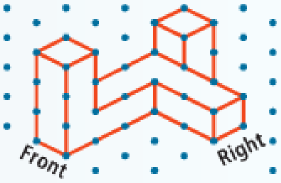


Exercises

5. The net below is for a number cube. What are the three sums of the numbers on opposite surfaces of the cube?



6. Make an orthographic drawing for the isometric drawing at the right. Assume there are no hidden cubes.



1-2 Points, Lines, and Planes

Quick Review

A **point** indicates a location and has no size.

A **line** is represented by a straight path that extends in two opposite directions without end and has no thickness.

A **plane** is represented by a flat surface that extends without end and has no thickness.

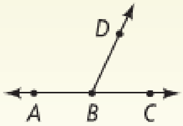
Points that lie on the same line are **collinear points**.

Points and lines in the same plane are **coplanar**.

Segments and **rays** are parts of lines.

Example

Name all the segments and rays in the figure.

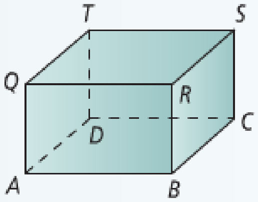


Segments: $\overline{AB}$, $\overline{AC}$, $\overline{BC}$, and $\overline{BD}$

Rays: $\overrightarrow{BA}$, $\overrightarrow{CA}$ or $\overrightarrow{CB}$, $\overrightarrow{AC}$ or $\overrightarrow{AB}$, $\overrightarrow{BC}$, and $\overrightarrow{BD}$

Exercises

Use the figure below for Exercises 7–9.



7. Name two intersecting lines.

8. Name the intersection of planes *QRBA* and *TSRQ*.

9. Name three noncollinear points.

Determine whether the statement is *true* or *false*. Explain your reasoning.

10. Two points are always collinear.

11. $\overrightarrow{LM}$ and $\overrightarrow{ML}$ are the same ray.

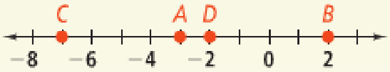
1-3 Measuring Segments

Quick Review

The **distance** between two points is the length of the segment connecting those points. Segments with the same length are **congruent segments**. A **midpoint** of a segment divides the segment into two congruent segments.

Example

Are $\overline{AB}$ and $\overline{CD}$ congruent?



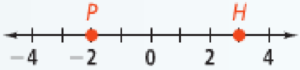
$AB = |-3 - 2| = |-5| = 5$

$CD = |-7 - (-2)| = |-5| = 5$

$AB = CD$, so $\overline{AB} \cong \overline{CD}$.

Exercises

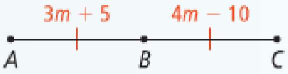
For Exercises 12 and 13, use the number line below.



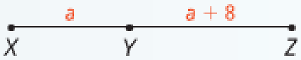
12. Find two possible coordinates of Q such that $PQ = 5$.

13. Use the number line above. Find the coordinate of the midpoint of $\overline{PH}$.

14. Find the value of m.



15. If $XZ = 50$, what are XY and YZ?



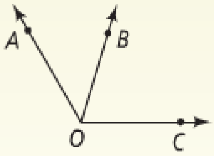
1-4 Measuring Angles

Quick Review

Two rays with the same endpoint form an **angle**. The endpoint is the **vertex** of the angle. You can classify angles as acute, right, obtuse, or straight. Angles with the same measure are **congruent angles**.

Example

If $m\angle AOB = 47$ and $m\angle BOC = 73$, find $m\angle AOC$.

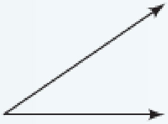


$$\begin{aligned} m\angle AOC &= m\angle AOB + m\angle BOC \\ &= 47 + 73 \\ &= 120 \end{aligned}$$

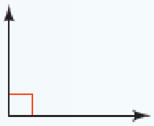
Exercises

Classify each angle as *acute, right, obtuse,* or *straight*.

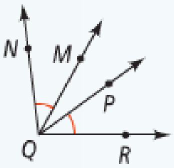
16.



17.



Use the diagram below for Exercises 18 and 19.



18. If $m\angle MQR = 61$ and $m\angle MQP = 25$, find $m\angle PQR$.

19. If $m\angle NQM = 2x + 8$ and $m\angle PQR = x + 22$, find the value of x.

1-5 Exploring Angle Pairs

Quick Review

Some pairs of angles have special names.

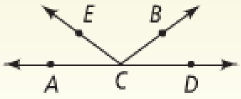
- **Adjacent angles:** coplanar angles with a common side, a common vertex, and no common interior points
- **Vertical angles:** sides are opposite rays
- **Complementary angles:** measures have a sum of 90
- **Supplementary angles:** measures have a sum of 180
- **Linear pair:** adjacent angles with noncommon sides as opposite rays

Angles of a linear pair are supplementary.

Example

Are ∠*ACE* and ∠*BCD* vertical angles? Explain.

No. They have only one set of sides with opposite rays.



Exercises

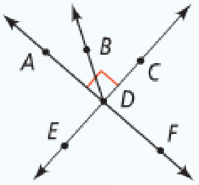
Name a pair of each of the following.

20. complementary angles

21. supplementary angles

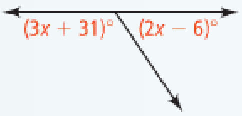
22. vertical angles

23. linear pair

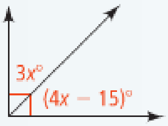


Find the value of *x*.

24.



25.



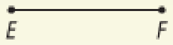
1-6 Basic Constructions

Quick Review

Construction is the process of making geometric figures using a **compass** and a **straightedge**. Four basic constructions involve congruent segments, congruent angles, and bisectors of segments and angles.

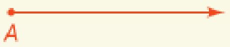
Example

Construct $\overline{AB}$ congruent to $\overline{EF}$.



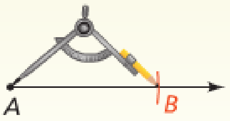
Step 1

Draw a ray with endpoint *A*.



Step 2

Open the compass to the length of $\overline{EF}$. Keep that compass setting and put the compass point on point *A*. Draw an arc that intersects the ray. Label the point of intersection *B*.

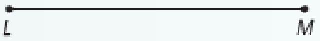


Exercises

26. Use a protractor to draw a 73° angle. Then construct an angle congruent to it.

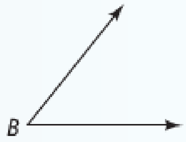
27. Use a protractor to draw a 60° angle. Then construct the bisector of the angle.

28. Sketch $\overline{LM}$ on paper. Construct a line segment congruent to $\overline{LM}$. Then construct the perpendicular bisector of your line segment.



29. a. Sketch ∠*B* on paper. Construct an angle congruent to ∠*B*.

b. Construct the bisector of your angle from part (a).



1-7 Midpoint and Distance in the Coordinate Plane

Quick Review

You can find the coordinates of the midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ using the **Midpoint Formula.**

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

You can find the distance d between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ using the **Distance Formula.**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example

$\overline{GH}$ has endpoints $G(-11, 6)$ and $H(3, 4)$. What are the coordinates of its midpoint M?

$$x\text{-coordinate} = \frac{-11 + 3}{2} = -4$$

$$y\text{-coordinate} = \frac{6 + 4}{2} = 5$$

The coordinates of the midpoint of $\overline{GH}$ are $M(-4, 5)$.

Exercises

Find the distance between the points to the nearest tenth.

30. $A(-1, 5)$, $B(0, 4)$

31. $C(-1, -1)$, $D(6, 2)$

32. $E(-7, 0)$, $F(5, 8)$

$\overline{AB}$ has endpoints $A(-3, 2)$ and $B(3, -2)$.

33. Find the coordinates of the midpoint of $\overline{AB}$.

34. Find AB to the nearest tenth.

M is the midpoint of $\overline{JK}$. Find the coordinates of K.

35. $J(-8, 4)$, $M(-1, 1)$

36. $J(9, -5)$, $M(5, -2)$

37. $J(0, 11)$, $M(-3, 2)$

1-8 Perimeter, Circumference, and Area

Quick Review

The perimeter P of a polygon is the sum of the lengths of its sides. Circles have a circumference C. The area A of a polygon or a circle is the number of square units it encloses.

Square: $P = 4s$; $A = s^2$

Rectangle: $P = 2b + 2h$; $A = bh$

Triangle: $P = a + b + c$; $A = \frac{1}{2}bh$

Circle: $C = \pi d$ or $C = 2\pi r$; $A = \pi r^2$

Example

Find the perimeter and area of a rectangle with $b = 12$ m and $h = 8$ m.

$$P = 2b + 2h \qquad A = bh$$

$$= 2(12) + 2(8) \qquad = 12 \cdot 8$$

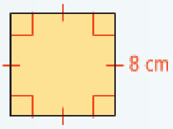
$$= 40 \qquad = 96$$

The perimeter is 40 m and the area is 96 m^2.

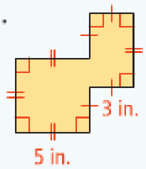
Exercises

Find the perimeter and area of each figure.

38.



39.



Find the circumference and the area for each circle in terms of π.

40. $r = 3$ in.

41. $d = 15$ m

1 Chapter Test

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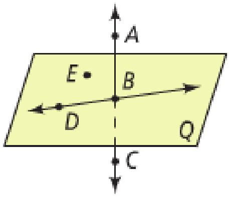
Do you know HOW?

1. Draw a net for a cube.

2. Draw an obtuse $\angle ABC$. Use a compass and a straightedge to bisect the angle.

Use the figure for Exercises 3–6.



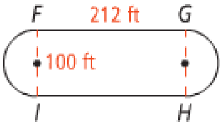
3. Name three collinear points.

4. Name four coplanar points.

5. What is the intersection of $\overleftrightarrow{AC}$ and plane Q?

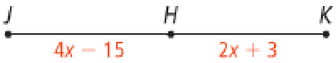
6. How many planes contain the given line and point?
   - a. $\overleftrightarrow{DB}$ and point A
   - b. $\overleftrightarrow{BD}$ and point E
   - c. $\overleftrightarrow{AC}$ and point D
   - d. $\overleftrightarrow{EB}$ and point C

7. The running track at the right is a rectangle with a half circle on each end. $\overline{FI}$ and $\overline{GH}$ are diameters. Find the area inside the track to the nearest tenth.



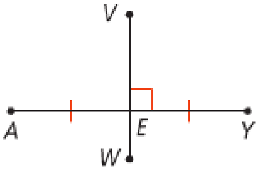
8. **Algebra** $M(x, y)$ is the midpoint of $\overline{CD}$ with endpoints $C(5, 9)$ and $D(17, 29)$.
   - a. Find the values of x and y.
   - b. Show that $MC = MD$.

9. **Algebra** If $JK = 48$, find the value of x.



10. To the nearest tenth, find the perimeter of $\triangle ABC$ with vertices $A(-2, -2)$, $B(0, 5)$, and $C(3, -1)$.

Use the figure to complete each statement.



11. $\overline{VW}$ is the $\underline{\ ?\ }$ of $\overline{AY}$.

12. If $EY = 3.5$, then $AY = \underline{\ ?\ }$.

13. $AE = \frac{1}{2}\underline{\ ?\ }$

14. $\underline{\ ?\ }$ is the midpoint of $\underline{\ ?\ }$.

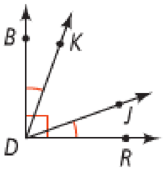
For the given dimensions, find the area of each figure. If necessary, round to the nearest hundredth.

15. rectangle with base 4 m and height 2 cm

16. square with side length 3.5 in.

17. circle with diameter 9 cm

Algebra **Find the value of the variable.**



18. $m\angle BDK = 3x + 4$, $m\angle JDR = 5x - 10$

19. $m\angle BDJ = 7y + 2$, $m\angle JDR = 2y + 7$

Do you UNDERSTAND?

Determine whether each statement is *always*, *sometimes*, or *never* true.

20. $\overrightarrow{LJ}$ and $\overrightarrow{TJ}$ are opposite rays.

21. Angles that form a linear pair are supplementary.

22. The intersection of two planes is a point.

23. Complementary angles are congruent.

24. **Writing** Explain why it is useful to have more than one way to name an angle.

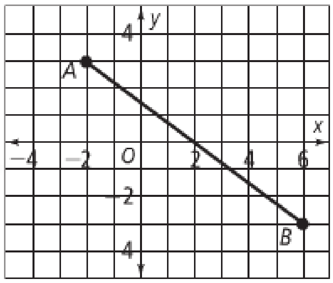
25. You have 30 yd^2 of carpet. You want to install carpeting in a room that is 20 ft long and 15 ft wide. Do you have enough carpet? Explain.

1 Cumulative Test Prep

TIPS FOR SUCCESS

Some questions ask you to find a distance using coordinate geometry. Read the sample question at the right. Then follow the tips to answer it.

What is the distance from the midpoint of $\overline{AB}$ to endpoint B?



A $\sqrt{10}$

B 5

C 10

D 100

TIP 2

Use the Distance Formula to find the length of the segment.

TIP 1

The midpoint divides the segment into two congruent segments that are each half of the total length.

Think It Through

Find AB using the Distance Formula.

$$AB = \sqrt{(-2-6)^2 + [3-(-3)]^2}$$
$$= \sqrt{100}$$
$$= 10$$

The distance from the midpoint of $\overline{AB}$ to endpoint B is $\frac{1}{2}AB$, or 5. The correct answer is B.

Vocabulary Builder

As you solve test items, you must understand the meanings of mathematical terms. Match each term with its mathematical meaning.

A. segment

B. angle bisector

C. construction

D. net

E. congruent angles

I. angles with the same measure

II. a two-dimensional diagram of a three-dimensional figure

III. the part of a line consisting of two endpoints and all points between

IV. a ray that divides an angle into two congruent angles

V. a geometric figure made using a straightedge and compass

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. Points A, B, C, D, and E are collinear. A is to the right of B, E is to the right of D, and B is to the left of C. Which of the following is NOT a possible arrangement of the points from left to right?

A D, B, A, E, C

B D, B, A, C, E

C B, D, E, C, A

D B, A, E, C, D

2. A square and a rectangle have equal area. If the rectangle is 36 cm by 25 cm, what is the perimeter of the square?

F 30 cm

G 60 cm

H 120 cm

I 900 cm

3. Which construction requires drawing only one arc with a compass?

Ⓐ constructing congruent segments

Ⓑ constructing congruent angles

Ⓒ constructing the perpendicular bisector

Ⓓ constructing the angle bisector

4. Rick paints the four walls in a room that is 12 ft long and 10 ft wide. The ceiling in the room is 8 ft from the floor. The doorway is 3 ft by 7 ft, and the window is 6 ft by 5 ft. If Rick does NOT paint the doorway or window, what is the approximate area that he paints?

Ⓕ 301 ft^2

Ⓖ 322 ft^2

Ⓗ 331 ft^2

Ⓘ 352 ft^2

5. If $\angle A$ and $\angle B$ are supplementary angles, what angle relationship between $\angle A$ and $\angle B$ CANNOT be true?

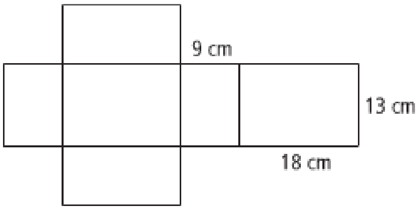
Ⓐ $\angle A$ and $\angle B$ are right angles.

Ⓑ $\angle A$ and $\angle B$ are adjacent angles.

Ⓒ $\angle A$ and $\angle B$ are complementary angles.

Ⓓ $\angle A$ and $\angle B$ are congruent angles.

6. A net for a small rectangular gift box is shown below. What is the total area of the net?



Ⓕ 468 cm^2

Ⓖ 782 cm^2

Ⓗ 1026 cm^2

Ⓘ 2106 cm^2

7. The measure of an angle is 12 less than twice the measure of its supplement. What is the measure of the angle?

Ⓐ 28

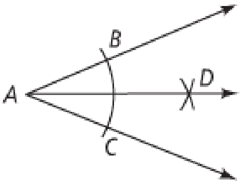
Ⓑ 34

Ⓒ 64

Ⓓ 116

8. Given: $\angle A$

What is the second step in constructing the angle bisector of $\angle A$?



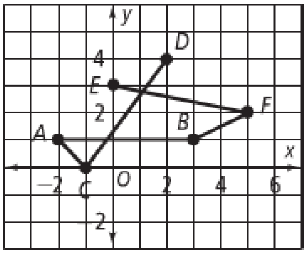
Ⓕ Draw $\overrightarrow{AD}$.

Ⓖ From points B and C, draw equal arcs that intersect at D.

Ⓗ Draw a line segment connecting points B and C.

Ⓘ From point A, draw an arc that intersects the sides of the angle at points B and C.

9. Which two segments are congruent?



Ⓐ $\overline{AB}$ and $\overline{CD}$

Ⓑ $\overline{EF}$ and $\overline{AB}$

Ⓒ $\overline{CD}$ and $\overline{EF}$

Ⓓ $\overline{AC}$ and $\overline{BF}$

10. Which postulate most closely resembles the Angle Addition Postulate?

Ⓕ Ruler Postulate

Ⓖ Protractor Postulate

Ⓗ Segment Addition Postulate

Ⓘ Area Addition Postulate

11. What is the length of the segment with endpoints $A(1, 7)$ and $B(-3, -1)$?

Ⓐ $\sqrt{40}$

Ⓑ 8

Ⓒ $\sqrt{80}$

Ⓓ 40

12. Which of the following does NOT extend forever in at least one direction?

- (F) line
- (G) plane
- (H) ray
- (I) segment

13. Which point is exactly 5 units from $(-10, 4)$?

- (A) $(-6, -7)$
- (B) $(-6, 7)$
- (C) $(6, -7)$
- (D) $(6, 7)$

GRIDDED RESPONSE

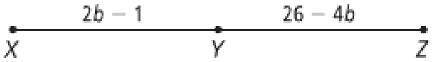
14. The measure of an angle is 78 less than the measure of its complement. What is the measure of the angle?

15. The face of a circular game token has an area of 10π cm^2. What is the diameter of the game token? Round to the nearest hundredth of a centimeter.

16. The measure of an angle is one third the measure of its supplement. What is the measure of the angle?

17. Bill's bike wheels have a 26-in. diameter. The odometer on his bike counts the number of times a wheel rotates during a trip. If the odometer counts 200 rotations during the trip from Bill's house to school, how many feet does Bill travel? Round to the nearest foot.

18. Y is the midpoint of $\overline{XZ}$. What is the value of b?



19. A rectangular garden has dimensions 8 ft by 16 ft. If you triple the length of each side of the garden, how many times greater than the original area will the resulting area be?

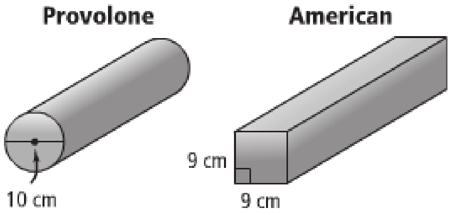
20. The sum of the measures of a complement and a supplement of an angle is 200. What is the measure of the angle?

21. What is the area in square units of a rectangle with vertices $(-2, 5)$, $(3, 5)$, $(3, -1)$, and $(-2, -1)$?

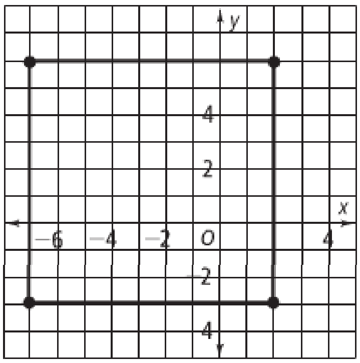
22. $\overline{AB}$ has endpoints $A(-4, 5)$ and $B(3, 5)$. What is the x-coordinate of a point C such that B is the midpoint of $\overline{AC}$?

Short Response

23. The two blocks of cheese shown below are cut into slices of equal thickness. If the cheese sells at the same cost per slice, which type of cheese slice is the better deal? Explain your reasoning.

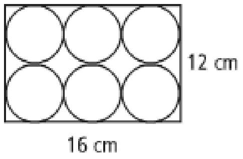


24. Copy the graph below. Connect the midpoints of the sides of the square consecutively. What is the perimeter of the new square? Show your work.



Extended Response

25. A packaging company wants to fit 6 energy-drink cans in a cardboard box, as shown below. The bottom of each can is a circle with an area of 9π cm^2.



a. What is the area of the bottom of the box?

b. What is the total area taken up by the bottoms of the cans? Round to the nearest hundredth.

c. Will the cans fit in the bottom of the box? Explain.